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Artificial dissipation model applied to Euler equations for analysis of supersonic flow around a geometric configurations ramp and diffusor type

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ABSTRACT

Very High Temperature Gas Cooled Reactors - VHTGRs are studied by several research groups for the development of advanced reactors that can meet the world's growing energy demand. The analysis of the flow of helium coolant around the various geometries at the core of these reactors through computational fluid dynamics techniques is an essential tool in the development of conceptual designs of nuclear power plants that provide added security. This analysis suggests a close analogy with aeronautical cases widely studied using computational numerical techniques to solve systems of governing equations for the flow involved. The present work consists in using the DISSIPA2D_EULER code, to solve the Euler equations in a conservative form, in two-dimensional space employing a finite difference formulation for spatial discretization using the Euler method for explicit marching in time. The physical problem of supersonic flow of helium gas along a ramp and diffusor configurations is considered. For this, the Jameson and Mavriplis algorithm and the linear artificial dissipation model of Pulliam was implemented. A spatially variable time step is employed aiming to accelerate the convergence to the steady state solution. The main purpose of this work is obtained computational tools for flow analysis through the study the cited dissipation model and describe their characteristics in relation to the overall quality of the solution, as well as obtain preliminary results for the development of computational tools of dynamic analysis of helium gas flow in gas-cooled reactors.

***Keywords*: VHTGRs, Euler Equations, Artificial Dissipation Model.**

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1. INTRODUCTION

Work with numerical methods requires sensitivity on the part of the programmer regarding the suitability of the methods chosen to physical proposed situation. For simulations involving Computational Fluid Dynamics (CFD), it is necessary a previous knowledge of some key items about the type of flow and boundary conditions. However, a numerical scheme may present numerical instabilities associated with it, such as those that arise from the use of centered operators for spatial discretization, for example. One way to treat such instabilities is the introduction of artificial dissipation terms in order to provide the necessary stability in the numerical convergence scheme. [8].

Artificial dissipation operators are an important tool in working with numerical schemes of symmetric spatial discretization. These operators have the basic function of providing better treatment to the numerical instabilities inherent in the numerical method, present in the convergence process. In this way, these operators, distinguish different numerical damping types and produce appropriate damping quantities, allowing a better computational efficiency of the numerical scheme, resulting in more accurate solutions.

Symmetric schemes with artificial dissipation operators properly formulated can lead to satisfactory results in terms of quality and quantity of the generated solution. The Jameson and Mavriplis scheme [1] is a example of a structured, symmetrical scheme with good resolution, which can lead to satisfactory results using suitable dissipation operators. The scheme was developed for problems in structured grids of triangles, but its extension to rectangle meshes in straightforward.

Pulliam [12] presented an analysis between two artificial models used in symmetric algorithms with structured spatial discretization: both scalar and isotropic, being a linear and the other nonlinear. An implicit numerical scheme approximated factorized was used to perform numerical experiments in which dissipation artificial models were implemented in LHS (Left Hand Side), to increase the robustness and the convergence rate of the scheme.

For the application and understanding of techniques widely used in aerospace engineering to study the helium flow around geometries presents in the core of the VHTGR, through the analysis of pressure and Mach number contours, preliminary tests on geometries of smaller degree of complexity are needed, therefore, a ramp and diffusor configurations were adopted in this work by the simplicity in the visualization of the results. The present work is part of an extensive study that aims at the development of a computational tool for analysis of helium flow along a VHTGR channel, for better understanding of this event.

The paper analyzes the DISSIPA2D_EULER code [9], generated from the implementation artificial dissipations models linear (studied in this paper) and nonlinear of Pulliam [12] in a finite difference context and using a generalized coordinate system, applied to the solution of a helium gas supersonic flow along a ramp and diffusor configurations. Its extension to other geometries as well as other regime flows (subsonic, hypersonic) is straightforward. The Jameson and Mavriplis algorithm [1] was used to perform the numerical experiments. The Euler equations are solved and a spatially variable time step was implemented to accelerate the convergence. This technique has proved excellent gains in terms of convergence ration as reported in [6]. The results have demonstrated that the isotropic linear scalar model of Pulliam [12] yielded converged solutions, in both cases (ramp and diffusor).

More complete studies involving other different geometries and physical problems are aimed by this author with the intention of better highlighting the characteristics of these artificial dissipation model, aiming the extension to the study of transients in the VHTGR core.

2. MATERIALS AND METHODS

2.1. Euler Equations

The Euler equations describing the motion of fluids, and expressing mass conservation, momentum and energy to a non-viscous, not heat-conducting and compressible medium in the absence of external forces. Depending on the complexity of the geometry to be worked, can be transformed of cartesian coordinates to generalized curvilinear coordinates, where:

$$\tau = t, \ \xi = \xi(x,y,t) \ \ and \ \ \eta = \eta(x,y,t) \tag{1}$$

In generalized curvilinear coordinates, the Euler equations take the following form:

$$\partial_\tau \bar{Q} + \partial_\xi \bar{E} + \partial_\eta \bar{F} = 0 \tag{2}$$

where $\bar{Q}$ is the vector of conserved variables to a generalized curvilinear coordinate system, $\bar{E}$ and $\bar{F}$ are the Euler flux vectors in the ξ and η directions, respectively. The $\bar{Q}$, $\bar{E}$ and $\bar{F}$ vectors are described below:

$$\bar{Q} = J^{-1}\begin{Bmatrix} \rho \\ \rho u \\ \rho v \\ e \end{Bmatrix}, \bar{E} = J^{-1}\begin{Bmatrix} \rho U \\ \rho u U + \xi_x p \\ \rho v U + \xi_y p \\ U(e+p) - \xi_t p \end{Bmatrix}, \bar{F} = J^{-1}\begin{Bmatrix} \rho V \\ \rho u V + \eta_x p \\ \rho v V + \eta_y p \\ V(e+p) - \eta_t p \end{Bmatrix} \tag{3}$$

where ρ is the fluid density, u and v are the cartesian components of the velocity vector in the x and y directions, respectively; e is the total energy, U and V are the contravariant velocities and p is the static pressure.

The Jacobian of the coordinate transformation and the metric terms are defined as follows:

$$J = \frac{1}{x_\xi y_\eta - x_\eta y_\xi}, \quad \xi_x = J y_\eta, \quad \xi_y = -J x_\eta, \quad \eta_x = -J y_\xi, \ and \ \eta_y = J x_\eta \tag{4}$$

The contravariant velocities, U and V, to a stationary mesh, are defined as:

$$U = \xi_x u + \xi_y v \ and \ V = \eta_x u + \eta_y v \tag{5}$$

To the studied problem, the Euler equations were nondimensionalized in relation to the freestream density, ρ_∞, the freestream speed of sound, a_∞. Hence, the density is nondimensionalized in relation to the ρ_∞, the velocity components u and v are nondimensionalized in relation to the a_∞, the pressure and the total energy are nondimensionalized in relation to the product $\rho_\infty(a_\infty)^2$. The matrix system of Euler equations is closed with the state equation to a perfect gas $p = (\gamma - 1)[e - 0,5\rho(u^2 + v^2)]$, assuming the ideal gas hypothesis, where γ is the ratio of specific heats. The calculation domain is divided in a big number of the rectangular cells and the Equation (2) is applied for each cell (for this simulations, gamma for the helium gas was estimated in 1,67 with values for specific heats obtained in KTA [11]).

2.2. Jameson and Mavriplis algorithm

The spatial derivatives $\partial_\xi \bar{E}$ and $\partial_\eta \bar{F}$ can be approximated by central differences of second-order. Thus, Equation (2) can be rewritten as

$$\Delta\bar{Q} = -\Delta t\left(\delta_\xi \bar{E} + \delta_\eta \bar{F}\right) \tag{6}$$

where,

$$\delta_\xi \bar{E}_{i,j} = \frac{\bar{E}_{i+1,j} - \bar{E}_{i-1,j}}{2} \text{ and } \delta_\eta \bar{F}_{i,j} = \frac{\bar{F}_{i,j+1} - \bar{F}_{i,j-1}}{2} \tag{7}$$

Thus, Equation (6) shall be written as follows:

$$\bar{Q}^{n+1} = \bar{Q}^n - \Delta t\left(\frac{\bar{E}_{i+1,j} - \bar{E}_{i-1,j}}{2} + \frac{\bar{F}_{i,j+1} - \bar{F}_{i,j-1}}{2}\right) \tag{8}$$

The introduction of a dissipation operator "D" is required in order to ensure numerical stability, for example, in the case of odd-even uncoupling of solutions and nonlinear instabilities, such as shock waves. Then Equation (8) is rewritten as:

$$\bar{Q}^{n+1} = \bar{Q}^n - \Delta t\left(\frac{\bar{E}_{i+1,j} - \bar{E}_{i-1,j}}{2} + \frac{\bar{F}_{i,j+1} - \bar{F}_{i,j-1}}{2}\right) + D_\xi + D_\eta \tag{9}$$

where D_ξ and D_η are operators in the ξ and η directions, respectively.

2.3. Artificial dissipation operator

The artificial dissipation model studied in this work is based on the Pulliam [12], and involves the isotropic artificial dissipation linear model, to provide dissipation to the numerical scheme.

2.3.1. Isotropic scalar linear model of Pulliam (1986)

The isotropic scalar linear artificial dissipation model of Pulliam (1986) consists in providing uniform dissipation in the calculation domain and in the four's conservation equations, acting independently in each coordinate direction. In other words, the dissipation provided in the ξ direction takes into account a weighting over the information originating from the instabilities in the η direction with the same relative weight of the ξ direction, characterizing, hence, an isotropism of the model.

$$D_\xi = D_\xi^{(4)} = -\Delta t_{i,j}\varepsilon_E J^{-1}\left(\nabla_\xi \Delta_\xi\right)^2 J\bar{Q}^n \text{ and } D_\eta = D_\eta^{(4)} = -\Delta t_{i,j}\varepsilon_E J^{-1}\left(\nabla_\eta \Delta_\eta\right)^2 J\bar{Q}^n \quad (10)$$

The ε_E parameter has its value adjusted through numerical experimentation. Due to the fact of does not exist distinction between the instabilities resulting from uncoupled solutions and the instability resulting from the shock wave presence, for example, Pulliam (1986) suggests the use of the isotropic scalar nonlinear model.

2.4. Spatially variable time step

The basic idea of this procedure consists in keeping constant the CFL number in all computational domain, allowing, hence, the use of appropriated time steps to each specific mesh region during the convergence process.

$$\Delta t_{i,j} = CFL(\Delta s)_{i,j} / c_{i,j} \quad (11)$$

where CFL is the "Courant-Friedrichs-Lewy" number, to provide numerical stability to the scheme; $c_{i,j}$ is the maximum characteristic velocity of information transport in the calculation domain; and $(\Delta s)_{i,j}$ is a characteristic length of information transport. According to finite difference formalism, for a generalized curvilinear coordinates system, can be define $(\Delta s)_{i,j}=\Delta\xi=\Delta\eta=1$ and $c_{i,j}=m\acute{a}x\left(|U|+a\sqrt{\xi_x^2+\xi_y^2},|V|+a\sqrt{\eta_x^2+\eta_y^2}\right)_{i,j}$.

2.5. Initial and Boundary conditions

2.5.1. Initial conditions

Values of freestream flow are adopted for all properties as initial condition, in the whole calculation domain, to the physical problem studied in this work. The vector of conserved variables is expressed as follows [8]:

$$Q_\infty=\left\{1 \quad M_\infty\cos\theta \quad M_\infty\text{sen}\theta \quad \left[\frac{1}{\gamma(\gamma-1)}+\frac{M_\infty^2}{2}\right]\right\}^t \tag{12}$$

where M_∞ represents the freestream Mach number and θ is the angle of incidence of the flow upstream of the configuration.

2.5.2. Boundary conditions

The boundary conditions are basically of three types: solid wall, entrance and exit. Details of these implementations are available in [8] and [10].

2.6. Tool validation

The validation of the tool followed a standard of analysis considering aeronautical cases. Data obtained from Maciel (2007) through the implementation of the schemes of Harten (1983), Frink, Parikh e Pirzadeh (1991), Liou e Steffen (1993), Radespiel e Kroll (1995), all first-order precision in space, were used for the comparison with the solutions obtained in the present work with the scheme of Jameson and Mavriplis [1], for the pressure ratio on the wall of the ramp, in the case of Euler, through the DISSIPA2D_EULER code. The Euler equations in their conservative form, implemented through the use of a formulation of finite volumes (except the Jameson and Mavriplis [1] scheme) and structured spatial discretization in the two-dimensional space, were solved. The results obtained are shown in Fig. 1. In the specific case of the supersonic flow for the ramp configuration, the schemes present good behavior, with emphasis in the scheme of Harten (1983).

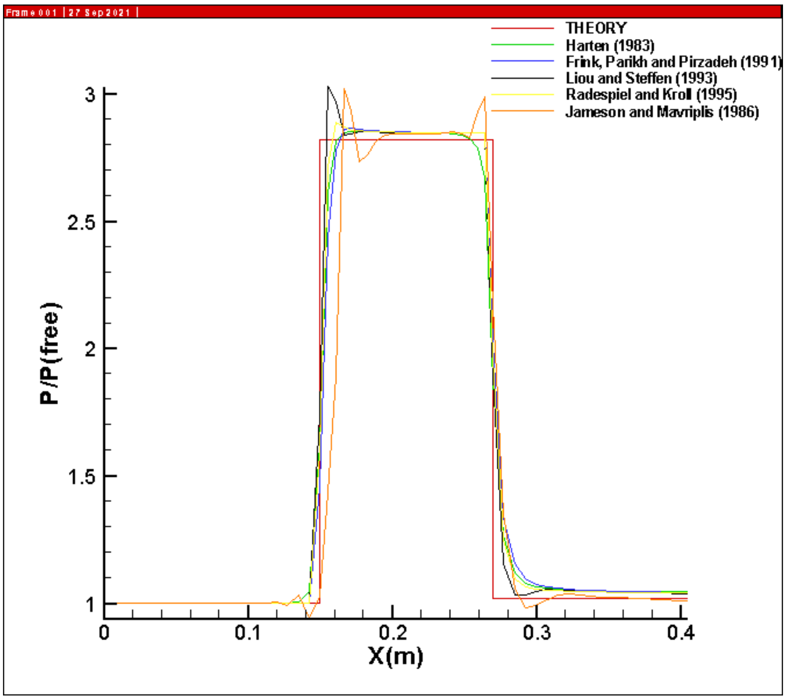

Figure 1: *The behavior of the scheme of Jameson and Mavriplis*

The behavior of the scheme of Jameson and Mavriplis [1] can be evidenced in Fig. 1. As can be observed, the pressure has a little oscillation in relation to the theoretical value, and, therefore, may indicate an error. However, the generated solution does not diverge from the theoretical value in order to compromise the quality of the method used, taking into account that a scheme with second-order in space inadequately treats oscillations in regions with high pressure gradients. This will happen when it comes to second-order precision schemes. Then, based on the comparisons and considering the particularities of each scheme, the DISSIPA2D_EULER code showed good behavior. After this verification, it was possible to follow the simulations for the remaining cases.

3. RESULTS AND DISCUSSION

In this study, the meshes generated algebraically to solve the Euler equations in shown in Fig. 2 and Fig. 3.

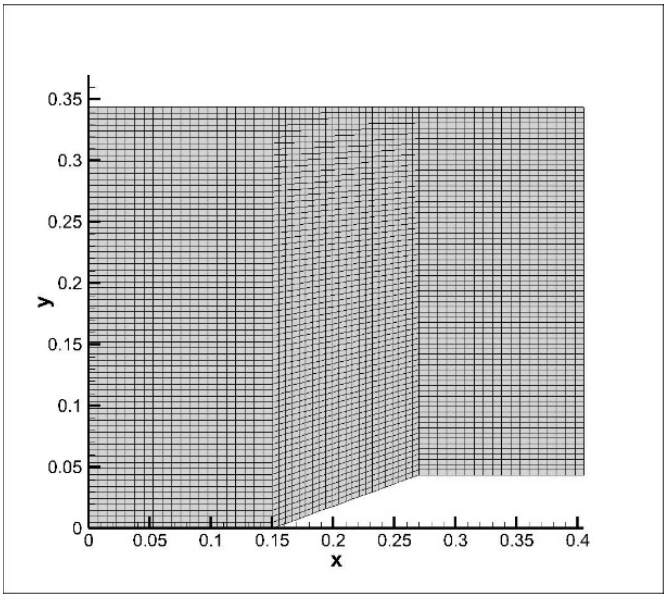

Figure 2: *Mesh for the ramp configuration*

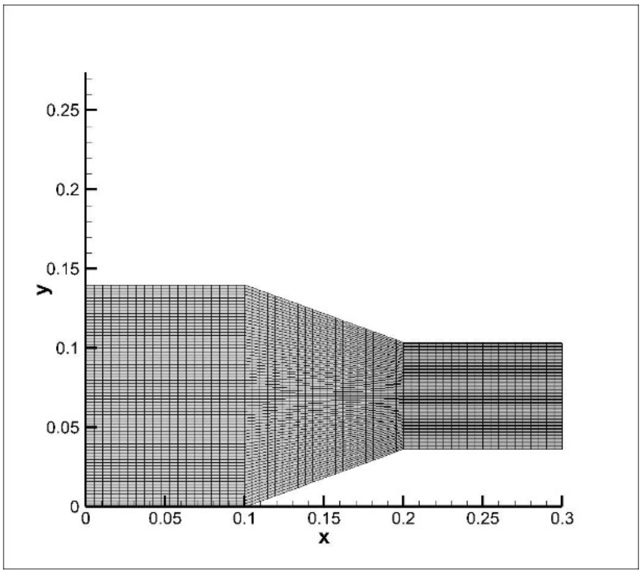

Figure 3: *Mesh for the diffuser configuration*

Tests were performed in an "INTEL CORE i5-4210U – 3.40GHz e and 8 Gbyte of RAM memory computer". For the simulation, the Mach number adopted was 2.0 (which characterizes a supersonic flow). The value used for CFL number in this works was 0.1, the γ used was 1,67 (for the helium gas) and for the attack angle the value adopted was 0,0°. The isotropic scalar linear model of Pulliam [12], object of this study, yielded converged results, with residue of the order of 10^{-7}.

3.1. The ramp physical problem

An algebraic mesh of 61x71 points was used to this problem. The Fig. 4 and 5 exhibit, respectively, pressure and Mach number contours generated by Jameson and Mavriplis scheme. For the purpose of show the behavior of the contours of pressure and Mach number were realized simulations with different values of ε_E. The variation of the parameter ε_E reflected directly in the final number of iterations, as can be seen in the Tab. 1. As ε_E was increased, decreased the number of iterations, which was expected, because the increase of ε_E implies more linear dissipation added to the system. Although not existing a standard value for ε_E, care must be taken in its growth, because determined ε_E values can directly affect the solution of the problem, in other words, exists a compromise between the solution and the value of ε_E.

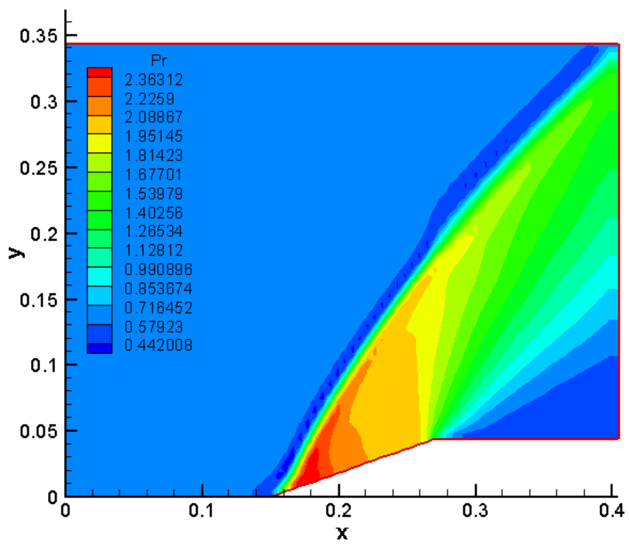

Figure 4: *Pressure contours for the ramp configuration*

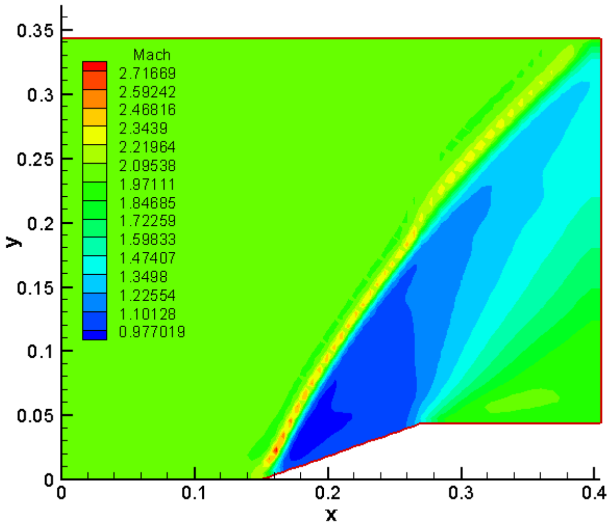

Figure 5: *Mach number contours for the ramp configuration*

It is possible to note loss of homogeneity in the ramp contours. Pre-shock oscillations are perceived. It should be noted that such behavior is typical of second-order algorithms and thus does not represent an error of implementation in the algorithm, which does not affect capture of the shock. Figures 4 and 5 show the pressure and Mach number contours for $\varepsilon_E = 45$.

Table 1: ε_E versus Iterations.

CFL	ε_E	Iterations
0,1	5	-
-	15	2446
-	25	2306
-	35	2243
-	45	2160
-	55	2111
-	65	2068
-	75	2011
-	85	1984
-	95	1955

3.2. The diffuser physical problem

An algebraic mesh of 61x71 points was used to this problem. After the simulations with the ramp mesh type, simulations were performed involving diffusor configuration. The Fig. 6 and 7 exhibits, respectively, pressure and Mach number contours generated for ε_E = 185.

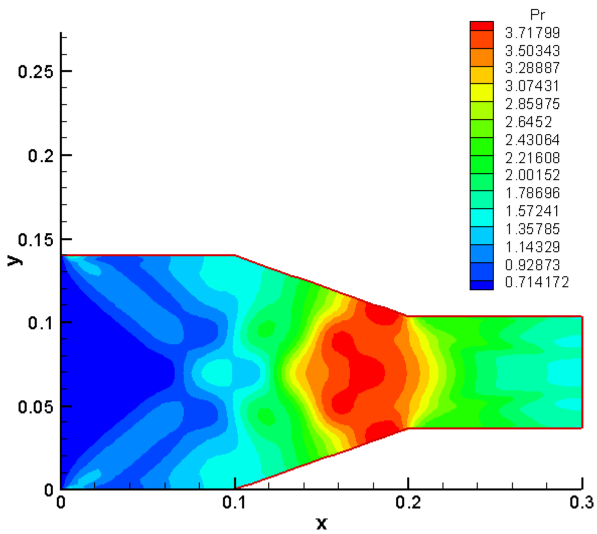

Figure 6: *Pressure contours for the diffuser configuration*

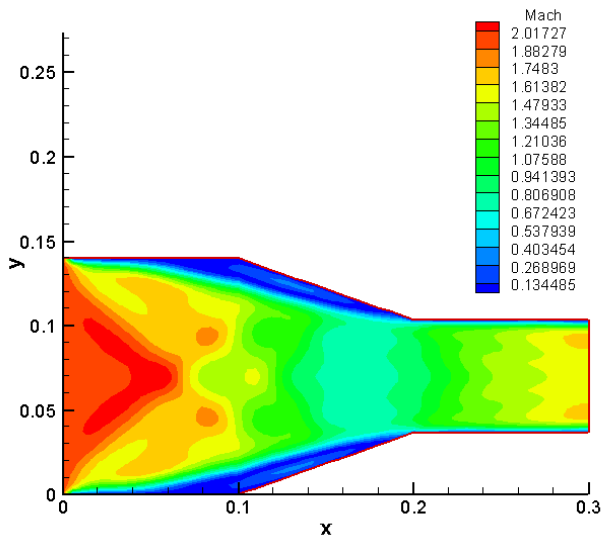

Figure 7: *Mach number contours for the diffuser configuration*

The contour view in the Fig. 7 shows the good properties of symmetry obtained with the linear model of Pulliam [12], as well as it is possible to notice a smaller loss of homogeneity when compared to the ramp case. The pressure and Mach number contours present good symmetry and the shock interference between the upper wall shock and the lower wall shock is well highlighted, important fact for the design of specific geometries and materials for a given component, which is typical during the development of components in nuclear power plants, for example. Other simulations can be realized, aiming better analysis of the contours.

4. CONCLUSIONS

The present work exhibits the DISSIPA2D_EULER code, generated through of numerical implementation of the Jameson e Mavriplis algorithm and the linear and nonlinear dissipation models of Pulliam applied to solve the Euler equations in two-dimensions. The physical problems of the supersonic flow along a ramp and diffuser were studied. The spatially variable time step was implemented to accelerate the convergence process. The results have demonstrated that the scalar linear model of Pulliam presents accurate solutions, and that Jameson and Mavriplis scheme may present more critical solutions, in relation to the other proposed schemes, however, without compromising the convergence of the scheme. In the both problems (ramp and diffuser), the isotropic scalar linear model yielded converged results. The quality of the solution related to the linear model of Pulliam, in this case, is related to parameter ε_E.

As conclusion, the Jameson and Mavriplis algorithm can assume critical values, but which are relatively acceptable, taking into account, that the discretization of the governing equations with central formulas is not dissipative: such formulas can lead to spurious oscillations in problems in which convection is dominant. It is also possible to verify that, with the appropriated estimate for ε_E parameter, other values of ε_E can be performed to find a better balance between ε_E and the quality of the solution generated. In this study it is important to highlight that computational numerical techniques are widely used in aeronautical and aerospace problems, can be adapted, with the proper implementation, for the study of the flow of refrigerant flow in the core of gas cooled reactors.

More studies, with other example-cases, will be performed by this author aiming to better detect the characteristics of the algorithms presented in this work.

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