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Albedo method applied for gamma radiation shielding

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ABSTRACT

Mathematical tools for calculating radiation shielding usually have difficult notations that could only be solved by computational methods. The Albedo's method applied for calculation of shielding proves to be an excellent substitute in determining the incident beam fractions that are reflected, absorbed and transmitted, avoiding the use of the transport equation and diffusion approximation that are extremely important in nuclear reactor designs and irradiation calculations. Based on the simple following of the radiation current path, the method can be characterized as a graphical and analytical solution. This work explores the Albedo's Method applied to 4-slab shielding with incidence of gamma radiation to an energy group compared to ANISN, computational method consecrated in the area of nuclear calculations.

Keywords: Albedo, Nuclear Reactor, Radiation shielding

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1. INTRODUCTION

When talking about mathematical tools within the area of Nuclear Engineering, it is possible to think about the classic ones that are treated in the course, such as the Transport Equation (linearized Boltzmann Equation) and the Diffusion Equation (an approximation of the Neutron Transport Equation) or computational techniques such as the Monte Carlo's Method (based on probabilities for simulating a phenomenon). Both the Transport Theory and the Monte Carlo's Method could be used when it comes to radiation shielding. However, the Albedo's Method has proven to be a great substitute over the years when applied to the same shielding calculations as it is a graphical and analytical solution in which the radiation current path is followed.

It is noteworthy that the Transport equation has great importance in Reactor and Shielding Physics because both problems deal with the interaction of neutron with matter. However, there are differences in the application of calculations in these two areas. While in Neutronics a few medium paths of neutrons are followed, in Shielding a longer path is followed. As a result, the Shielding project involves calculating the transport of complex systems, most of the time.

With the computational breakthrough, many numerical techniques were developed and along with them, solution methods for the Transport Equation as well as new computational codes that use new methods of solution, such as ANISN [11].

Finally, for more irregular three-dimensional geometries the only tool available is the Monte Carlo Method.

As seen above, the Albedo's Method proves to be both efficient and suitable for Neutronic and shielding calculations, where the latter will be the object of this research that seeks to revisit the method for shielding gamma radiation in 4 slabs and compare its efficiency with the established computational model ANISN.

1.1. Historical context of Albedo's method

In 1958, Alan M. Jacobs developed a theory of Albedo to two energy groups for calculating the reactivity temperature coefficient of nuclear reactors [1]. The agreement between theoretical and experimental data [2] confirmed the validity of the theory.

In 1991, an energy multigroup Albedo's Method for neutron was developed for thermal nuclear reactor calculations [2]. The agreement verified between theoretical and experimental methods confirms the validity of Albedo for calculations of effective neutron multiplication factors for the treated systems.

Between 1993 and 1995, works were presented [3-5] where the Albedo's Method was used for two energy groups to calculate the effective multiplication factor of neutrons in thermal reactors.

In 1996 the Albedo's Method for two energy groups was applied to calculate the effective multiplication factor of neutrons in fast reactors [6] and in the same year the application of the method to shielding neutron currents was expanded [7]. Again, the result obtained by the method was confronted with the diffusion predictions, and both showed whole agreement.

In 1998 and 1999 applications of the Albedo's Method to energy multigroup were developed for neutron shielding problems in present systems of two infinite plates and multiple infinite plates (slabs) [8] and finally applied to gamma radiation shielding [9].

And also, in 2000, a study of Albedo's method for gamma radiation shielding was published with results corroborated by ANISN's results [10].

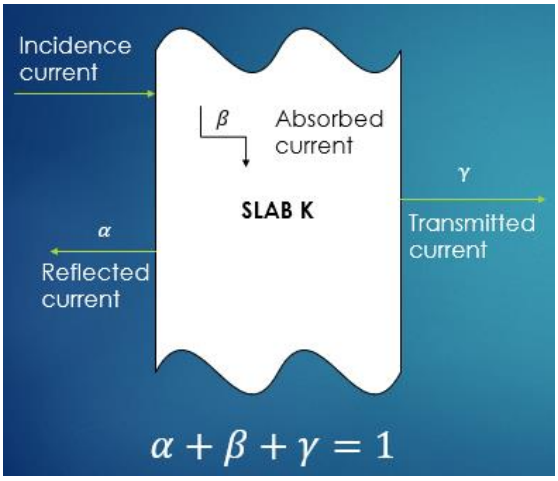
2. MATERIALS AND METHOD

The sentence that guides the Albedo's Method is "follow the radiation current path". However, at first, it is necessary to understand the concept of Albedo itself. Albedo is actually the so-called reflection coefficient that a given material or given object has intrinsically. To apply the method in radiation shielding it is necessary to know the coefficients of Reflection (Albedo), absorption and transmission of the material. When a radiation current falls over a certain object, part of it is reflected, absorbed and transmitted through the object. The reflection coefficient can be determined as the ratio of the current that was reflected ($J_{reflected}$) by the current that was incident on the material ($J_{incidence}$). It can be expressed mathematically as:

$$Albedo = \frac{J_{reflected}}{J_{incidence}} = \alpha \tag{1}$$

The following model can be schematized as shown in the figure below:

Figure 1 Slab's coefficients for absorption, transmission and reflection of incident radiation.

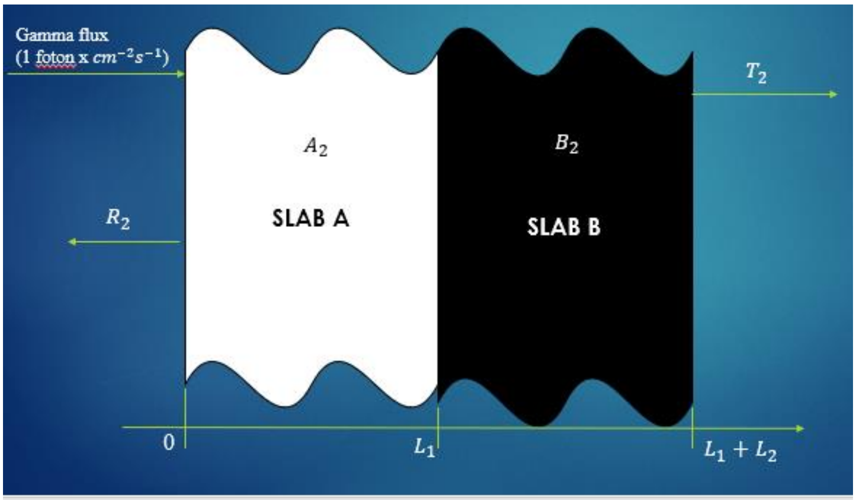


In the same way as it was defined in Eq. 1, it can be defined the absorption (β) and transmission (γ) coefficients of the board. An interesting relationship can already be deduced as shown in Fig. 1, the sum of all the coefficients on a plate equals to 1 due to the total incident radiation rate, assuming nothing is lost.

2.1. Albedo's method for 2 slabs (direct incidence)

In a bilaminate shielding model, where there are two slabs, it is possible to develop a mathematical method whose objective is to determinate the total reflected, absorbed and transmitted by the two slabs, applying the Albedo's Method, following the incident radiation path that can be schematized as follows bellow:

Figure 2: Absorption, Transmission and Reflection coefficients for two slabs scheme



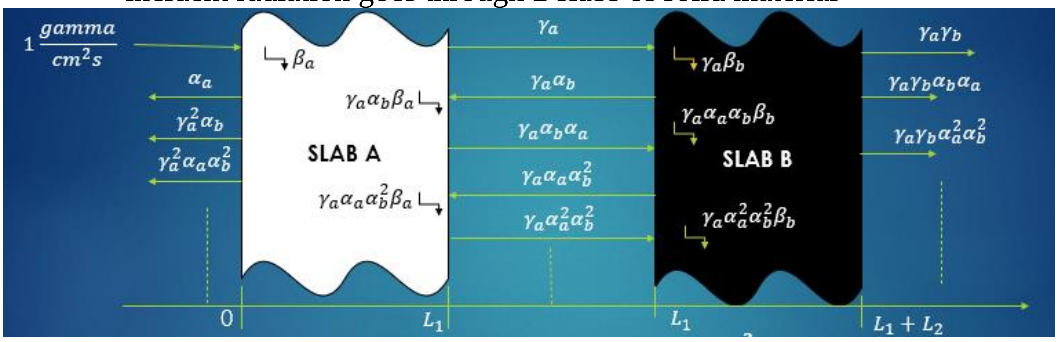
In the Fig. 2, the main notation is defined, where R_n is the reflection that happens in *n* slabs, T_n the transmission that happens for *n* slabs, K_n the total absorption on the slab K in a set of *n* slabs. A table can be defined with the respective coefficients of each slab as shown in the Tab. 1.

Table 1: Slab's coefficients

Coefficients	Slab A	Slab B
Reflection	α_a	α_b
Absorption	β_a	β_b
Transmission	γ_a	γ_b

As already mentioned, the path of the incident current must be followed by observing what happens between plates A and B, according to the Fig. 3:

Figure 3: Albedo's method for two slabs– Absorption, Transmission and Reflection when incident radiation goes through 2 slabs of solid material



According with Fig. 3, a current of $1\frac{gamma}{cm^2 s}$ is incident on slab A, a current rate is reflected (α_a), the other is absorbed (β_a) and the last one is transmitted to slab B (γ_a). This last current begins its incidence on slab B and goes through the same processes as slab A, a rate is reflected, other absorbed and the last rate transmitted. However, part of this reflection that reach the slab B, returns to slab A, where what is called "Ping Pong" of reflections between the slabs start to happen, as can be seen in the figure. Finally, each of these reflections that occur in this Ping Pong will generate contributions to the total reflection, absorption and transmission of the slabs. Therefore, a sum of all these contributions must be made to obtain the final values. It is worth remembering that this contribution event happens instantly. Now, it can be defined the reflection, absorption in each slab and transmission through the slab B according to the equations below:

$$R_2 = \alpha_a + \gamma_a^2\alpha_b + \cdots = \alpha_a + \gamma_a^2\alpha_b(1 + \alpha_a\alpha_b + \alpha_a^2\alpha_b^2 + \cdots) = \boldsymbol{\alpha_a + \frac{\gamma_a^2\alpha_b}{1 - \alpha_a\alpha_b}} \tag{2}$$

$$A_2 = \beta_a + \gamma_a\alpha_b\beta_a + \gamma_a\alpha_a\alpha_b^2\beta_a \ldots = \beta_a + \gamma_a\alpha_b\beta_a(1 + \alpha_a\alpha_b + \cdots) = \boldsymbol{\beta_a + \frac{\gamma_a\alpha_b\beta_a}{1 - \alpha_a\alpha_b}} \tag{3}$$

$$B_2 = \gamma_a\beta_b + \gamma_a\beta_b\alpha_a\alpha_b + \cdots = \gamma_a\beta_b(1 + \alpha_a\alpha_b + \cdots) = \boldsymbol{\frac{\gamma_a\beta_b}{1 - \alpha_a\alpha_b}} \tag{4}$$

$$T_2 = \gamma_a\gamma_b + \gamma_a\gamma_b\alpha_a\alpha_b + \cdots = \gamma_a\gamma_b(1 + \alpha_a\alpha_a + \cdots) = \boldsymbol{\frac{\gamma_a\gamma_b}{1 - \alpha_a\alpha_b}} \tag{5}$$

The equations (2) to (5) allow to get the analytical equations to reflection (Eq. 2), absorption (Eq. 3 and 4) and the total transmission (Eq. 5) in the two slabs model. Note that the sum of each term is equal to the sum of an infinite geometric progression with ratio smaller than 1 ($\alpha_a\alpha_b$) the sum converges to a value possible to be calculated as shown above.

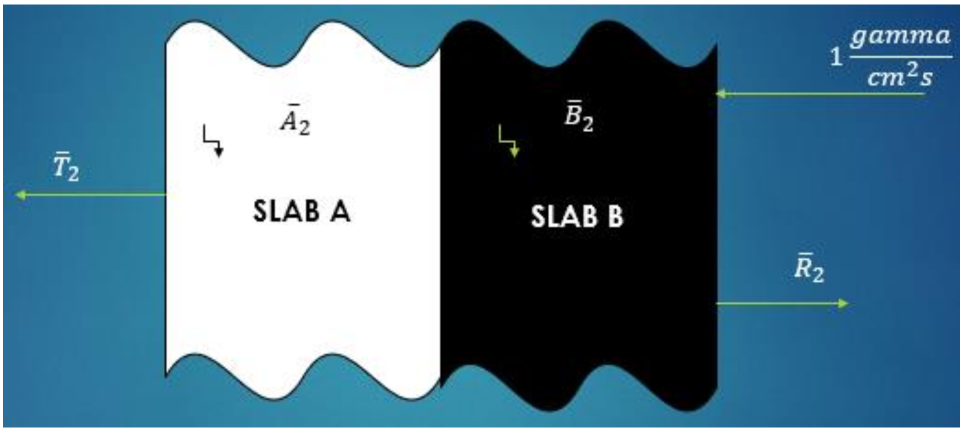
Before go on to the three-slab model, it is necessary to study what would happen if the gamma current were incident on plate B, instead of plate A. This process is called retro-incidence and will be discussed in the next topic.

2.2. Albedo's Method for 2 slabs (retro-incidence)

As mentioned in the previous topic, knowing the retro-incidence of the two-slab case is necessary so that there can be an analysis for the three-slab case.

For retro-incidence, the Albedo's Method is applied in the same way as for direct incidence, however, the gamma current incidence affects slab B first, as can be seen in the Fig. 4.

Figure 4: Retro-incidence of radiation in two slabs



In the Fig. 4 there is a new notation that will be used in this work. $\bar{R}_n$ represents the reflection current of the retro-incidence for *n* slabs, $\bar{T}_n$ is the transmission current for *n* slabs and $\bar{K}_n$ is the absorption on the slab K due to *n* slabs. Applying again the Albedo's Method:

$$\bar{R}_2 = \alpha_b + \frac{\gamma_b^2 \alpha_a}{1 - \alpha_a \alpha_b} \tag{6}$$

$$\bar{A}_2 = \frac{\gamma_b \beta_a}{1 - \alpha_a \alpha_b} \tag{7}$$

$$\bar{B}_2 = \beta_b + \frac{\gamma_b \alpha_a \beta_b}{1 - \alpha_a \alpha_b} \tag{8}$$

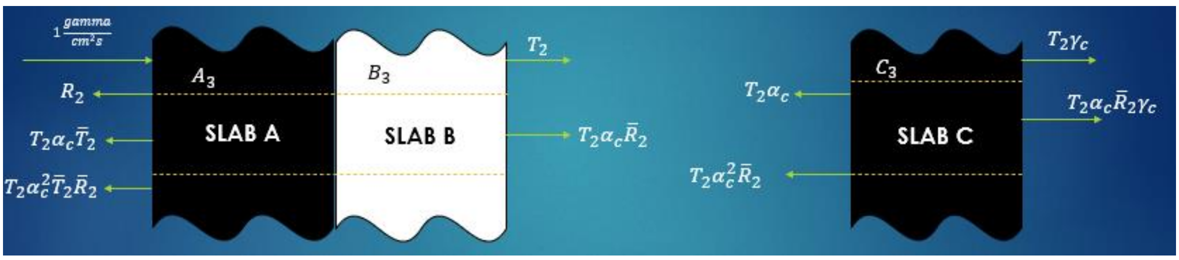
$$\bar{T}_2 = T_2 \tag{9}$$

A curious fact is that for two slabs, the transmission current doesn't change for direct incidence and retro-incidence as can be seen on the Eq. 9, but the reflection (Eq 6) and the absorption on each (Eq. 6, Eq.7 and Eq.8) slab do so. Now, there's the possibility to develop the method for the three slabs case.

2.3. Albedo's Method for 3 slabs (direct incidence)

For the 3 slabs case, the analysis for applying the method remains similar as in the topics above, however at this point there will be need to know the how the retro-incidence in two layers works as schematizing in the Fig. 5.

Figure 5: Albedo's method for 3 slabs- Absorption, Transmission and Reflection when incident radiation goes through 3 slabs of solid material.



Knowing the behave of the current in the case of two slabs, it can be expanded by adding one more plate and follow the current path as the previous one and considering the knowledge of the retro-incidence that occurs on the two slabs, it just needs to add one more slab and see what is happening as can be seen in the Fig.5. Applying Albedo's method, the following equations are obtained:

$$R_3 = R_2 + \frac{T_2 \alpha_c \bar{T}_2}{1 - \alpha_c \bar{R}_2} \tag{10}$$

$$A_3 = A_2 + \frac{T_2 \alpha_c \bar{A}_2}{1 - \alpha_c \bar{R}_2} \tag{11}$$

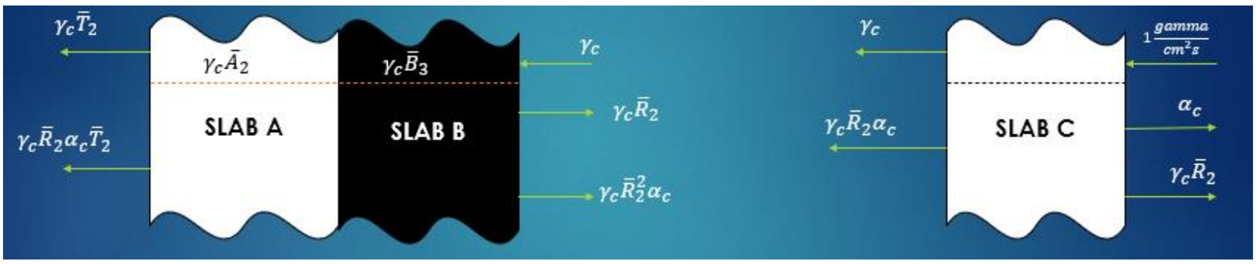
$$B_3 = B_2 + \frac{T_2 \alpha_c \bar{B}_2}{1 - \alpha_c \bar{R}_2} \tag{12}$$

$$C_3 = \frac{T_2 \beta_c}{1 - \alpha_c \bar{R}_2} \tag{13}$$

$$T_3 = \frac{T_2 \gamma_c}{1 - \alpha_c \bar{R}_2} \tag{14}$$

In order to achieve the objective of this work, the use of the four-slab method is sought, and for that, it is necessary to know the retro-incidence in the three-slab model, which can be developed as seen so far. To exemplify, the retro-incidence equations for three slabs will be shown below.

Figure 6: Three slabs case (retro-incidence)



$$\bar{R}_3 = \alpha_3 + \frac{\gamma_c^2 \bar{R}_2}{1 - \alpha_c \bar{R}_2} \tag{15}$$

$$\bar{A}_3 = \frac{\gamma_c \bar{A}_2}{1 - \alpha_c \bar{R}_2} \tag{16}$$

$$\bar{B}_3 = \frac{\gamma_c \bar{B}_2}{1 - \alpha_c \bar{R}_2} \tag{17}$$

$$\bar{C}_3 = \beta_3 + \frac{\gamma_c \bar{R}_2 \beta_c}{1 - \alpha_c \bar{R}_2} \tag{18}$$

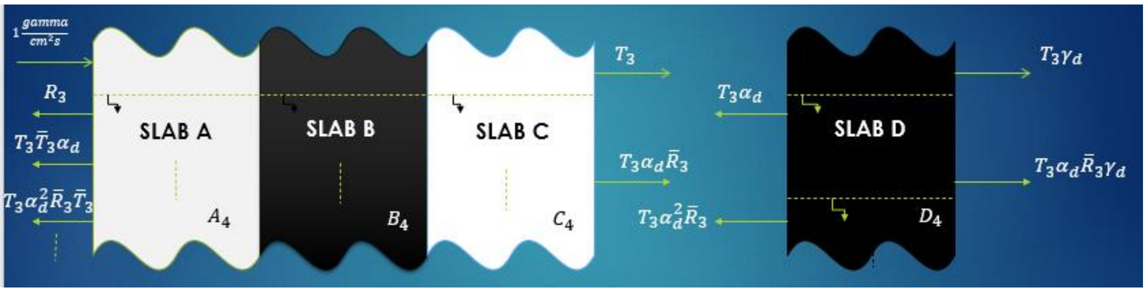
$$\bar{T}_3 = \frac{\gamma_c \bar{T}_2}{1 - \alpha_c \bar{R}_2} \tag{19}$$

With the retro-incidence equations of the three-slabs case, it's possible to know the four-slabs case.

2.4. Albedo's method for 4 slabs

For this case, the method remains the same. Following the current incident path on slab A and knowing the incidence of the three-slab model and its retro-incidence, it is possible to reach the reflection, absorption in each slab and the direct transmission that occurs through the plates. The Fig.7 shows the schematic layout of the model.

Figure 7: Albedo's method for 4 slabs



The reflection, absorption and transmission equations are obtained in each slab, making the sum of each contribution of retro-incidences that occur:

$$R_4 = R_3 + \frac{T_3 \bar{T}_3 \alpha_d}{1 - \alpha_d \bar{R}_3} \tag{20}$$

$$A_4 = A_3 + \frac{T_3 \alpha_d \bar{A}_3}{1 - \alpha_d \bar{R}_3} \tag{21}$$

$$B_4 = B_3 + \frac{T_3 \alpha_d \bar{B}_3}{1 - \alpha_d \bar{R}_3} \tag{22}$$

$$C_4 = C_3 + \frac{T_3 \alpha_d \bar{C}_3}{1 - \alpha_d \bar{R}_3} \tag{23}$$

$$D_4 = \frac{T_3 \beta_d}{1 - \alpha_d \bar{R}_3} \tag{24}$$

$$T_4 = \frac{T_3 \gamma_d}{1 - \alpha_d \bar{R}_3} \tag{25}$$

According to the equations (20) to (25), it was possible to apply the Albedo's Method to determine the gamma radiation shielding. To validate the method, ANISN data in previous works [9] with slabs of Iron, Aluminum, Lead and Nickel were used as reference.

According to this theoretical development, it is possible to notice that for the application of the method in four slabs, it is necessary to know the shielding of two and three slabs respectively. With this, the equations were carefully deduced and, according to their development, an Excel spreadsheet (Tab.2) was set up in order to obtain data from slabs with direct incidence and retro-incidence for two and three slabs. At the end, another table is set up (Tab.3) to obtain the data from the four slabs and with that, the validity of the data obtained is confirmed, adding the reflection, absorption and transmission data in order to obtain the initial incident current of $1 \frac{gamma}{cm^2 s}$.

3. RESULTS AND DISCUSSION

The Tab. 2 bellow shows the data of the slabs of Iron, Aluminum, Lead and Nickel given by ANISN.

Table 2: Slabs data given by ANISN

Materials	Reflection (α)	Absorption (β)	Transmission (γ)
Iron	4,28749E-01	1,89929E-01	3,81323E-01
Aluminum	2,54313E-01	1,79234E-02	7,27766E-01
Lead	2,66305E-01	7,33375E-01	3,21085E-04
Nickel	4,33738E-01	2,28709E-01	3,37553E-01

The Tab. 3 shows the data obtained by albedo's method algorithm for the shielding of 4-slabs composed by Iron, Aluminum, Lead and Nickel respectively.

Table 3: Albedo's method for 4-slabs

Four slabs					
Reflection	Absorption (Fe)	Absorption (Al)	Absorption (Pb)	Absorption (Ni)	Transmission
5,00143E-01	2,25489E-01	9,99329E-03	2,64304E-01	2,99170E-05	4,41547E-05

With the data above, obtained using the Albedo method, the comparison is made with Damaso´s method [9] and the ANISN code, as shown in Tab.4.

Table 4: Comparison between the Albedo method, the Damaso´s method and the ANISN code.

Data	Albedo Method	Damaso's method	ANISN code
Reflection	0,500143	0,500143	0,500142
Absorption (Fe)	0,225489	0,225489	0,225489
Absorption (Al)	0,0099933	0,0099933	0,00999332
Absorption (Pb)	0,264304	0,264304	0,264304
Absorption (Ni)	2,99170E-05	2,99170E-05	2,99169E-05
Transmission	4,41547E-05	4,41547E-05	4,41545E-05
Total	1,00000	1,00000	1,00000

The results provided by the Albedo's Method compared to data from [9] and the ANISN's deterministic transport code shown the efficiency of the method presented here with much similarity between the data.

4. CONCLUSION

The study that this work sought to bring was to evaluate the efficiency of the Albedo's method in shielding gamma radiation to one energy group in a system composed of four shielding plates, comparing it with other published works and with a consecrated deterministic method, the ANISN. The validity of the method is then demonstrated by comparing results that use the transport equation to determine the solution. It is also demonstrated that the method overcomes the limitation of the restrictions that are imposed by the approximation of the diffusion equation, being possible its application for photon shielding.

With regard to what was exposed above, one can understand the didactic and academic value in which the method is presented both in previous works and in this one. Due to its simple and fast application, with only the need for knowledge of shielding characteristics and initial incidence current, the method shows its main advantage in the treatment of data by presenting itself in an analytical and graphic solution of simple manipulation of the data, making the method a valuable mathematical tool.

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