Theoretical estimate of relative depth-doses for the asymmetric and spherical ^{106}Ru/^{106}Rh COC-type plaque used in radiotherapy

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ABSTRACT

In this work we present results of relative dose rate along central and lateral axis of the ^{106}Ru/^{106}Rh COC model plaque used for the treatment of tumors close to the optical nerve. The calculations were based on the beta-point dose function and are compared with an available data obtained by means of Monte Carlo simulations. Even with the limitations of the method, it was capable to reproduce the general trend of the dose rates around this kind of plaque and a maximum difference of -18% with MC was obtained. Despite the limitations of the method, the results may be used as a guide to future experimental and theoretical studies on this applicator, since there is a lack of available data in literature.

Keywords: beta particles, asymmetric beta ophthalmic applicators, relative depth-dose, ^{106}Ru/^{106}Rh.

1. INTRODUCTION

Over the last decades, applicators containing the pure beta emitters ^{106}Ru/^{106}Rh in secular equilibrium (parent ^{106}Ru with half-life of 373.6 days, and daughter ^{106}Rh with half-life of 29.8 seconds) are used in ophthalmic brachytherapy to the treatment of ocular malignancies, mainly melanomas and retinoblastomas [1]. However, a non-accurate dosimetry of these sources plays an important drawback in their use.

The difficulties in their dosimetry arise from the short dimensions involved and the steep fall-off of the dose rate with distance, and in some cases the geometric shape of the plaques may be an additional source of uncertainties. In this scenario calculation methods assume a special importance and there are basically only two ways to estimate the dose rates around these plaques. First, using Monte Carlo simulations (MC) that allow to handle with complex geometry of the sources, several absorbing media and may give results with great accuracy, but may be necessary a large time of computation, which is not desirable in a daily clinical use. Second, using analytical and numerical methods that give results in a negligible time of computation as compared with MC, but with less accuracy and apply only to simple geometries and homogeneous medium.

In this work we use this second approach to estimate the relative dose rates along the central and lateral axis of the curved and asymmetric COC-type plaque (manufactured by Eckert & Ziegler BEBIG, GmbH, Berlin, Germany) that emits beta particles from the ^{106}Ru/^{106}Rh radionuclides. This plaque contains a cut-out section designed to treat tumors close to the optical nerve, and this asymmetry makes still more harder the dosimetry and may explain the lack of theoretical and experimental dosimetric data on it. The results of dose rates along central axis of the COC plaque obtained by means of the analytical and numerical method described in this work are in a moderate accordance with the unique available MC-MCNPX simulations [2] (maximum difference of -18% at 4 mm depth). And when the results of this work are compared with the data from the manufacturer a maximum difference of -14% at 4 mm depth is found.

2. MATERIALS AND METHODS

There are two general theoretical ways to estimate the dose rates around beta applicators. First, by means of MC simulations, that is possible to simulate various geometries and media, and obtain results with a good accuracy, but may consume a large time of programming and computation and its implementation in a daily clinical routine may not be feasible. Second, using analytical and numerical methods, that are based on the beta point-kernel dose function integration, and may give accurate results of dose distributions in a negligible time of computation as compared with MC simulation, but apply only for water as an absorbing medium and to simple geometries. In this work this last approach was used and a numerical code was developed to estimate the dose rates along the central and lateral axes of the COC model applicator and in the following we describe this method.

The dose rate due to a beta-point source at a distance ξ is given by [3, 4]

$$J(\xi) = \frac{B}{(\rho v \xi)^2} \left\{ c \left[1 - \frac{\rho v \xi}{c} \exp\left(1 - \frac{\rho v \xi}{c} \right) \right] + \rho v \xi exp(1 - \rho v \xi) - \rho v \xi \left(1 - \frac{\rho v \xi}{2} - \frac{f}{2} \right) \right\}, \tag{1}$$

where ρ is the density of the medium, considered water-equivalent; v is the apparent coefficient of absorption, c and f are dimensionless parameters and B is a normalizing constant given by:

$$B = 0.046 \, \rho^2 v^3 \, E_\beta \, \alpha, \tag{2}$$

in which E_β is the mean beta energy per disintegration, and

$$\alpha^{-1} = 3c^2 - (c^2 - 1) \exp(1) + (3 + f) \exp(1 - f) - 4 \exp\left(1 - \frac{f}{2} \right). \tag{3}$$

In equations (1) and (3) the terms containing the quantity f come from a refinement proposed in ref. [4] in order to encompass the data not considered in ref. [3]. The expression of the beta-point dose function was obtained in an empirical way, but in a physical basis. The term within the bracket accounts for the energy absorbed from the un-scattered component of the beta particles and its value is zero at

$$\xi \geq {^c\!/_{\rho v}};$$ (4)

the second term accounts for the scattered component of the beta particles, and the inverse square attenuation law is present in the term ξ^2 at the denominator of (1). The third term in (1) was introduced by Vynckier and Wambersie [4] to comprise a set of new data not used by Loevinger [3], where

$$\xi \geq {^f\!/_{\rho v}}$$ (5)

are the distances from which the dose due to beta particles is zero. It should be noticed that the parameters v, c and f are unique for each nuclide.

For an extended source with the radioisotope uniformly distributed on its surface the dose rate is obtained by integration of the function (1) over its surface,

$$\dot{D} = a_S \iint J(\xi) \, dS,$$ (6)

where a_S is the surface activity and dS is the area element. Provided that the source has spherical symmetry (apart from the cut-out section, as shown in Figure 1, bottom) it is useful to use spherical coordinates, so that dS depends on the constant radius of curvature R; the azimuthal angle θ in the *xy*-plane measured from the x-axis ($0 \leq \theta \leq 2\pi$), and the polar angle φ measured from the positive *z*-axis ($0 \leq \varphi \leq \varphi_{max}$, where φ_{max} depends on the size of the plaque). Results of the application of this method have already been published for curved and symmetric applicators containing [106]Ru/[106]Rh radionuclides [5, 6].

Figure 1: *The geometry of the ^{106}Ru COC model plaque. Top, the spherical coordinates used in calculations with depth increasing from its center to the origin of the z-axis; bottom, a plane view depict of the plaque showing the cut-out section aligned with the x-axis.*
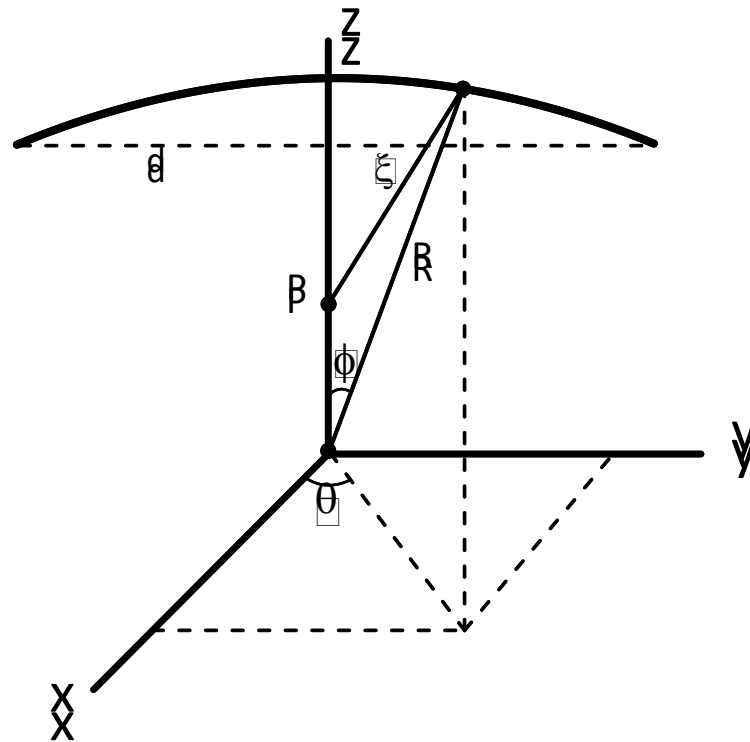
COC model plaque
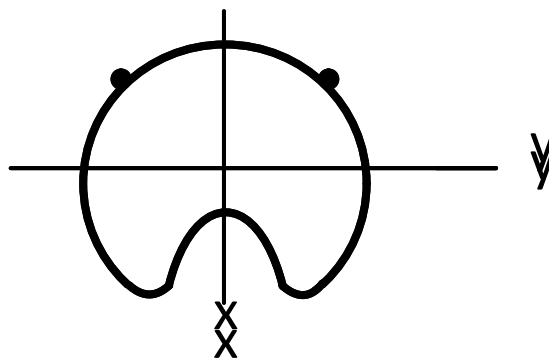
3. RESULTS AND DISCUSSION

In equations (1) up to (6) the absorbed dose rate $J(\xi)$ and the constant B are expressed in Gy/(MBq.h); ξ is expressed in centimeters, and dS in square centimeters. The surface activity is assumed to be 6 MBq/cm^2, although the results of calculations do not depend on it since we are interested in the relative dose rates; the density of the medium is 1 g/cm^3, and the coefficient of absorption is 3.57 cm^2/g. The average beta-particle energy per disintegration is 1.43 MeV, and the values of the parameters c and f are, respectively, 0.88 and 5.07. The plaque has a radius of curvature $R = 1.4$ cm and an active diameter $d = 2.27$ cm (data from the manufacturer).

A code written in Fortran programming language was used to implement the numerical integration of the Loevinger function, equations (1) up to (6). In this code the presence of the cut-out section was initially ignored, as if the source was symmetric. Then, an instruction within the code makes the program to skip the points inside the cut-out section, which are the points in the region bounded by the curves:

$$y = \pm\sqrt{r_0^2 - (x - x_0)^2}, \tag{7}$$

for $0.115 \leq x \leq 0.535$, and

$$y = \pm r_0, \tag{8}$$

for $0.535 < x \leq 1.135$, with x, y in centimeters and $x_0 = 0.535$ cm, $r_0 = 0.42$ cm (values based in the data provided in the manufacturer's manual).

In Figure 2 is depicted the results of dose rates normalized at 1mm depth plotted against the depth along z-axis, and in order to make a comparison it is also shown the unique available results of MC-MCNPX simulations [2] and data from the manufacturer (taken from [2]). The analytical/numerical calculations of this work disagree with the MC and certificate results by a maximum difference, respectively, of -18% and -14% at 4 mm depth (at a given depth calculated as the difference between our result and the result of ref. [2], relative to the result of ref. [2]). This disagreement may be assigned to the various simplifying hypotheses adopted. For example, the plaque (apart from the cut-out section) is considered spherically symmetric; the plaque is formed only by radioactive material; the encapsulation of radioactive material is not considered, and the

medium is homogeneous and constituted only of water. It should be noticed that the results obtained with MC codes may also present various sources of uncertainties, and the calibration certificate of this plaque comes with a declared uncertainty of ±20% in the dose rates along its central axis.

Figure 2: *The dose rate as a function of depth for the ^{106}Ru COC model plaque. Results are normalized at 1 mm depth. Squares, this work; circles, MC simulations [2]; triangles, data from calibration certificate (taken from [2]).*
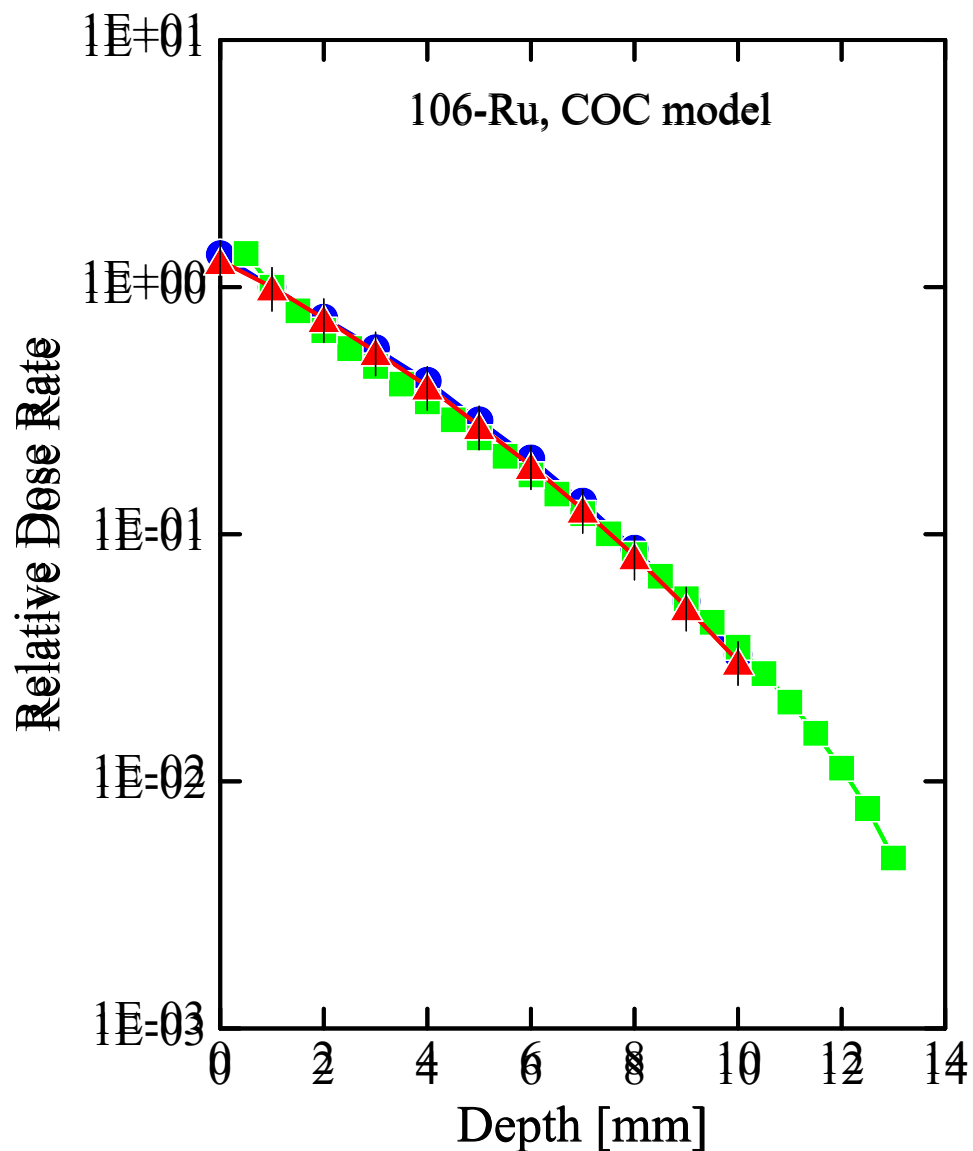
Finally, in Figure 3 is shown the dose rates along the lateral *x*-axis normalized at the point (0, 0, 1 mm), and a clearly effect of the cut-out section on the lateral dose rates can be seen.

Figure 3: *The relative dose rates as a function of the lateral distance x and at the depths of 1 up to 8 mm for the* ^{106}Ru *COC model plaque.*
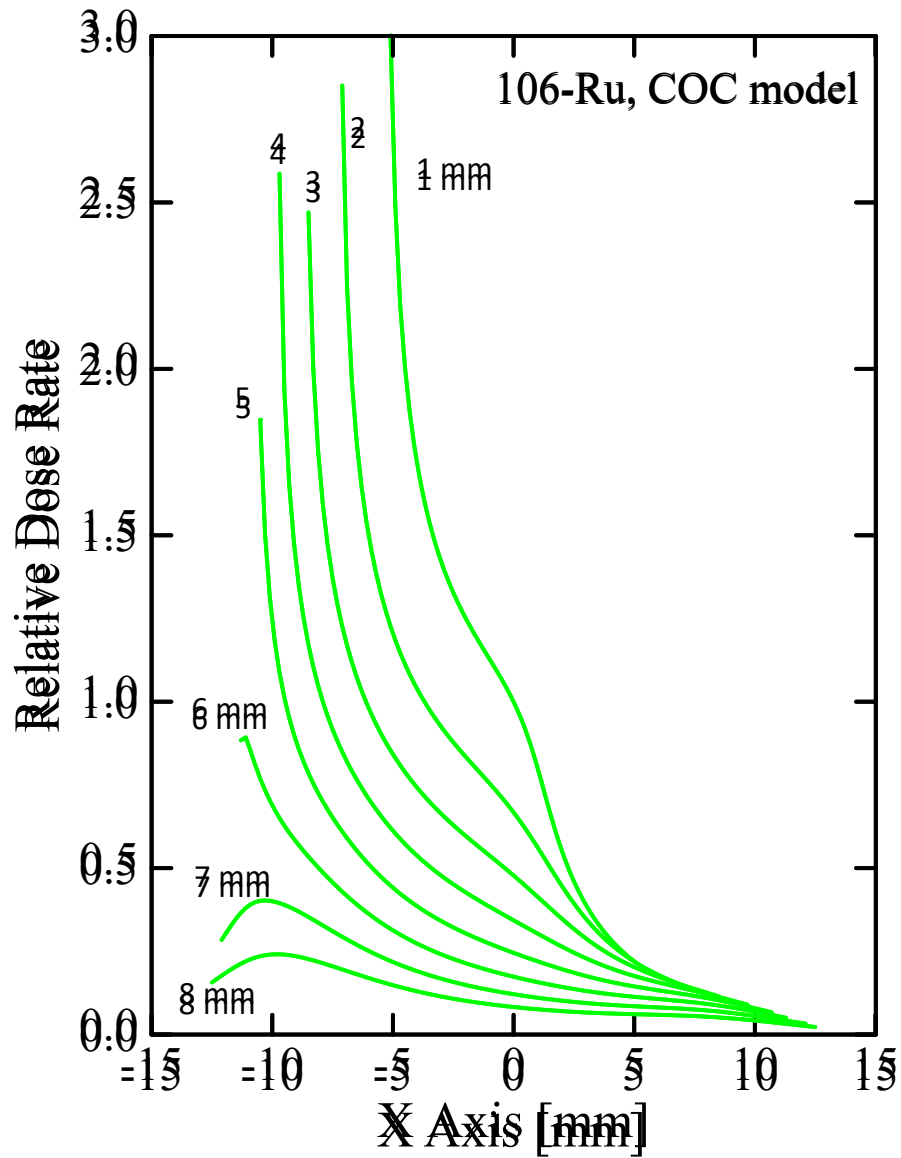
4. CONCLUSION

In this work we estimate the relative dose rate along central and lateral axis of ruthenium/rhodium COC-type applicator used in brachytherapy for the treatment of tumors close to the optical nerve and a comparison is made with the unique available MC simulations. The calculation was based on the numerical integration of the beta-point dose function. In spite of the limitations of the method used it may be used as a base for other experimental and theoretical studies on this plaque, provided that its asymmetry and the short distances involved make calculations and measurements of dose rates around this type of applicator very difficult and may explain the very low quantity of available data. At least, the results shown here may be used as guide and as an indicator of an inferior limit of the relative depth-dose for the ^{106}Ru/^{106}Rh COC-type plaque.

ACKNOWLEDGMENT

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