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Application of the piecewise constant function approximation method to modified point kinetics

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Abstract: This work uses the numerical method known as Piecewise Constant Approximation, PCA, to solve the equations of modified point kinetics for six groups of delayed neutron precursors. The modified point kinetics corresponds to the point kinetics model without considering the approximation for the derivative of the logarithm of the neutron current. Applying the PCA method approximates the reactivity function to continuous piecewise functions, and the resulting system of first-order differential equations can be solved exactly in each time partition. For validation, numerical simulations are carried out for the cases of constant reactivity, step type and time-varying reactivity, ramp type, and the results are compared with those obtained by the finite difference method. Quantitative analysis of the results shows that the PCA method can efficiently obtain good results for modified point kinetics.

Keywords: reactivity, point kinetics, PCA, relaxation time.

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Aplicação do Método de Aproximação por Funções Constantes por Partes na Cinética Pontual Modificada

Resumo: O presente trabalho utiliza o método numérico conhecido como "Piecewise Constant Approximation", PCA, na solução das equações da cinética pontual modificada para seis grupos de precursores de nêutrons atrasados. A cinética pontual modificada corresponde ao modelo da cinética pontual sem considerar a aproximação para a derivada do logaritmo da corrente. Aplicando-se o método PCA, a função reatividade é aproximada para funções contínuas por partes, e o sistema de equações diferenciais de primeira ordem resultante pode ser resolvido de maneira exata em cada partição do tempo. Para validação, são feitas simulações numéricas para os casos de reatividade constante, tipo degrau, e reatividade variante no tempo, tipo rampa e os resultados são comparados com os obtidos pelo método de diferenças finitas. A análise quantitativa dos resultados mostra que o método PCA pode ser empregado para obter, com eficiência, bons resultados também na cinética pontual modificada.

Palavras-chave: reatividade, cinética pontual, PCA, tempo de relaxação.

1. INTRODUCTION

By studying the behavior of the neutron population, it is possible to infer the stability of the chain reaction carried out in the reactor core. The central problem of nuclear reactor theory is determining the distribution of neutrons inside the reactor, as this will determine the rate at which the various nuclear reactions will take place inside the reactor. An important topic in reactor physics analysis is neutron kinetics, which makes it possible to predict the temporal behavior of the neutron population as a function of changes in reactivity. To determine the distribution of neutrons inside the reactor, the transport process, which is the movement of neutrons as they move around, often interacting with atomic nuclei through scattering and eventually being absorbed or escaping from the reactor, must be studied. The time-dependent neutron transport equation provides an accurate description of the behavior of the neutron distribution varying with time. Still, the solution of this equation is very complex, even for the stationary case. The neutron diffusion model is based on approximations from neutron transport theory and leads to the neutron diffusion equation, which, although not exact, the solution provided is obtained in a simple way and can be used in various cases of interest, satisfactorily describing the time-dependent neutron distribution. One of the neutron transport theories approximating that leads to the diffusion model is Fick's Law, which relates the neutron current density to the scalar flux. In the so-called point kinetics model, obtained from the diffusion model with an energy group, it is hypothesized that the spatial dependence of the neutron flux can be described from a single mode - the fundamental mode. With this proposition, it is possible to remove the spatial dependence of the diffusion model, arriving at a description involving only time-dependent ordinary differential equations [1]. The point kinetics equations are one of the most essential models in nuclear engineering. They have been the subject of several studies and applications for understanding neutron

dynamics and its effects. There are scientific papers in the literature dating from the 1950s to the present day that use different methodologies to approach point kinetics (e.g., [2], [3], [4], [5], [6], [7], [8], [9], [10], [11], [12], [13], [14], [15], [16], [17], [18], [19], [20], [21], [22], [23], [24], [25], [26], [27], [28], [29], [30]). In [20] it was developed the PCA method, "Piecewise Constant Approximation", which is based on approximations of the source and reactivity functions by piecewise continuous functions, efficiently calculating the solution of the point kinetics equations. In [26], a modified point kinetics model was proposed that considers the P1 equation without neglecting the derivative of the neutron current density instead of adopting the approximation given by Fick's Law, which results in the classical point kinetics equations. In [26] it was applied the traditional numerical method of finite differences to solve the equations of modified point kinetics. Modified point kinetics can be interpreted as a particular case of the fractional point kinetics model developed by [25].

This work aims to obtain the solution of the modified point kinetics equations deduced by [26] and [30] using the PCA method proposed by [20]. The methodology's application is validated by comparing it with the results obtained using the finite difference method. Quantitative analyses are made for the cases of constant reactivity, step type (prompt-subcritical, prompt-critical, and prompt-supercritical), and time-varying reactivity, ramp type.

The next section presents the mathematical development to implement the PCA method in modified point kinetics. In section 3, the proposed method is tested on two types of transients, typically used to check the responses of methods based on the point kinetics model. The final section presents the concluding analysis and future recommendations.

2. MATERIALS AND METHODS

2.1. Modified Point Kinetics and Point Kinetics

The modified point kinetics equations can be derived from the time-dependent neutron transport equations, as demonstrated in detail by [26] and later by [30]. These equations are given by:

$$\tau\frac{d^2n(t)}{dt^2}+\left[1+\tau\left(\frac{1}{l}-\frac{1-\beta}{\Lambda}\right)\right]\frac{dn(t)}{dt}=\frac{\rho(t)-\beta}{\Lambda}n(t)+\sum_{i=1}^{m}\lambda_i\left(C_i(t)+\tau\frac{dC_i(t)}{dt}\right) \quad (1)$$

$$\frac{dC_i(t)}{dt}=\frac{\beta_i}{\Lambda}n(t)-\lambda_iC_i(t) \quad i=1,2,\dots,m. \quad (2)$$

Where $n(t)$ is the neutron population; Λ is the neutron generation time; $\rho(t)$ is the time-dependent reactivity function; $l={f_a}^{-1}=(\upsilon\Sigma_a)^{-1}$ is the average prompt neutron lifetime; m is the total number of delayed neutron precursor groups; β_i is the fraction of delayed neutrons of precursor group i; β is the total fraction of delayed neutrons; $C_i(t)$ is the concentration of delayed neutron of precursor group i; λ_i is the decay constant of precursor group i. The parameter τ is the relaxation time (Espinosa-Paredes et al., 2011). $\tau^{-1}=f_d$ is defined as the transport frequency and is related to the rate of neutron transport reactions. The modified point kinetics model essentially consists of not neglecting the time derivative for the neutron current density in the equations that eventually lead to the neutron diffusion model, i.e., it is a model that modifies the diffusion approximation.

The point kinetics equations can be deduced in a similar way to the modified kinetics equations, as shown by [26] and [30], as long as the additional simplification of neglecting the time derivative for the neutron current density is taken into account, which leads to the

classic diffusion approximation. If we consider this simplification, the parameter τ becomes null, and Eq.1 falls back to the well-known point kinetics equation:

$$\frac{dn(t)}{dt} = \frac{\rho(t) - \beta}{\Lambda} n(t) + \sum_{i=1}^{m} \lambda_i C_i(t) \tag{3}$$

2.2. PCA Method in Modified Point Kinetics

In [20] the PCA method was proposed to solve the system of coupled ordinary differential equations of classical point kinetics, Eqs. 2 and 3. The method approximates the source and reactivity functions by piecewise continuous functions, transforming the problem into a system of linear ordinary differential equations that can be solved exactly for each time step. In order to apply the PCA method to the modified point kinetics equations, Eq. 1 and Eq. 2, it is necessary to put these equations into a more convenient form. Initially, the following auxiliary term is defined:

$$\Lambda_a = \left[1 + \tau\left(\frac{1}{l} - \frac{1-\beta}{\Lambda}\right)\right]\Lambda \tag{4}$$

moreover, substituting it into Eq. 1, we get:

$$\tau\frac{d^2n(t)}{dt^2} + \frac{\Lambda_a}{\Lambda}\frac{dn(t)}{dt} = \frac{\rho(t) - \beta}{\Lambda} n(t) + \sum_{i=1}^{m} \lambda_i \left(C_i(t) + \tau\frac{dC_i(t)}{dt}\right). \tag{5}$$

Making the following change of variables:

$$\frac{dn(t)}{dt} = y(t). \tag{6}$$

Substituting (6) into (5):

$$\tau \frac{dy(t)}{dt} + \frac{\Lambda_a}{\Lambda} y(t) = \frac{\rho(t) - \beta}{\Lambda} n(t) + \sum_{i=1}^{m} \lambda_i \left(C_i(t) + \tau \frac{dC_i(t)}{dt} \right). \tag{7}$$

Replacing (2) in (7) to eliminate the term with the derivative of the concentrations of the precursor groups:

$$\tau \frac{dy(t)}{dt} + \frac{\Lambda_a}{\Lambda} y(t) = \frac{\rho(t) - \beta}{\Lambda} n(t) + \sum_{i=1}^{m} \lambda_i \left(C_i(t) + \frac{\tau \beta_i}{\Lambda} n(t) - \tau \lambda_i C_i(t) \right). \tag{8}$$

Regrouping the terms of (8), we obtain the system of equations of modified point kinetics rewritten similarly to classical point kinetics:

$$\frac{dy(t)}{dt} = \left[\frac{\rho(t) - \beta + \tau \sum_{i=1}^{m} \lambda_i \beta_i}{\tau \Lambda} \right] n(t) - \frac{\Lambda_a}{\tau \Lambda} y(t) + \sum_{i=1}^{m} \left(\frac{\lambda_i - \tau \lambda_i^2}{\tau} \right) C_i(t), \tag{9}$$

$$\frac{dC_i(t)}{dt} = \frac{\beta_i}{\Lambda} n(t) - \lambda_i C_i(t) \quad i = 1, 2, \ldots, m\,. \tag{10}$$

Put the system of equations (9) and (10) into matrix form:

$$\frac{d\boldsymbol{x}(t)}{dt} = M(t)\boldsymbol{x}(t), \qquad (11)$$
$$\boldsymbol{x}(0) = \boldsymbol{x_0}\,.$$

Where the vector x(t) is defined as:

$$\boldsymbol{x}(t) = [n(t) \quad y(t) \quad C_1(t) \quad C_2(t) \quad \ldots \quad C_m(t)]^T,$$

$M(t)$ is the matrix defined as:

$$M(t) = \begin{bmatrix} 0 & 1 & 0 & 0 & \cdots & 0 \\ \dfrac{\rho(t)-\beta}{\tau\Lambda} + \displaystyle\sum_{i=1}^{m} \dfrac{\lambda_i\beta_i}{\Lambda} & -\dfrac{\Lambda_a}{\tau\Lambda} & \dfrac{\lambda_1}{\tau} - \lambda_1^2 & \dfrac{\lambda_2}{\tau} - \lambda_2^2 & \cdots & \dfrac{\lambda_m}{\tau} - \lambda_m^2 \\ \dfrac{\beta_1}{\Lambda} & 0 & -\lambda_1 & 0 & \cdots & 0 \\ \dfrac{\beta_2}{\Lambda} & 0 & 0 & -\lambda_2 & \ldots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \dfrac{\beta_m}{\Lambda} & 0 & 0 & 0 & \cdots & -\lambda_m \end{bmatrix},$$

and $\boldsymbol{x}(0)$ the initial condition vector:

$$\boldsymbol{x}(0) = \left[1 \quad \frac{\rho_0}{\Lambda_a} \quad \frac{\beta_1}{\lambda_1\Lambda} \quad \ldots \quad \frac{\beta_m}{\lambda_m\Lambda}\right]^T.$$

We proceed with the modified point kinetics equation, represented in matrix form, Eq. 11, which is similar to point kinetics, as developed by [20]. Piecewise constant functions approximate the reactivity function:

$$\rho(t) \approx \rho\left(\frac{t_i + t_{i+1}}{2}\right) = \rho_i, \qquad t_i \leq t \leq t_{i+1} \ . \tag{12}$$

The system of differential equations (11) is transformed into the following approximate system:

$$\frac{d\widehat{\boldsymbol{x}}}{dt} = M_i\widehat{\boldsymbol{x}}\,, \tag{13}$$

with $\widehat{\boldsymbol{x}}(t_i) = \widehat{\boldsymbol{x}}_{\boldsymbol{i}}$, , where $\boldsymbol{x}(t) \approx \widehat{\boldsymbol{x}}(t)$. Multiply both sides of the equation by the integrating factor $e^{-(M_i)t}$:

$$e^{-(M_i)t}\frac{d\widehat{\boldsymbol{x}}}{dt} = e^{-(M_i)t}(M_i)\widehat{\boldsymbol{x}}\,,$$

$$\frac{d}{dt}\left(e^{-(M_i)t}\widehat{\boldsymbol{x}}\right) = 0\,. \tag{14}$$

Integrate the two sides of equation (14) from $\boldsymbol{t_i}$ to $\boldsymbol{t_{i+1}}$, at time step $\boldsymbol{i}$:

$$\int_{t_i}^{t_{i+1}} d\left(e^{-(M_i)t}\widehat{\boldsymbol{x}}\right) = 0\,, \tag{15}$$

$$e^{-(M_i)t_{i+1}}\widehat{\boldsymbol{x}}_{\boldsymbol{i+1}} - e^{-(M_i)t_i}\widehat{\boldsymbol{x}}_{\boldsymbol{i}} = 0\,. \tag{16}$$

Developing equation (16) and making $h_i = t_{i+1} - t_i$, we arrive at:

$$\widehat{\boldsymbol{x}}_{\boldsymbol{i+1}} = e^{M_i h_i}\widehat{\boldsymbol{x}}_{\boldsymbol{i}}\,. \tag{17}$$

It is possible to diagonalize the matrix M_i in such a way as to make the calculation of equation (17) more efficient:

$$M_i = X_i D_i X_i^{-1}, \tag{18}$$

where X_i is the matrix constructed with the eigenvectors of the M_i matrix, X_i^{-1} is its respective inverse matrix, and D_i is the diagonal matrix, whose elements of the main diagonal are the eigenvalues associated with the M_i matrix. The eigenvalues and eigenvectors of the M_i matrix of the modified point kinetics can be determined by numerically calculating the roots of its respective characteristic polynomial. For the case of 6 groups of precursors, we have that:

$$det\begin{bmatrix} m_{11}-\lambda & m_{12} & m_{13} & \cdots & m_{18} \\ m_{21} & m_{22}-\lambda & m_{23} & \cdots & m_{28} \\ m_{31} & m_{32} & m_{33}-\lambda & \cdots & m_{38} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ m_{81} & m_{82} & m_{83} & \ldots & m_{88}-\lambda \end{bmatrix} = 0\,.$$

Finally, substituting the decomposition (18) into (17) gives:

$$\hat{\boldsymbol{x}}_{\boldsymbol{i+1}} = X_i e^{D_i h_i} X_i^{-1} \hat{\boldsymbol{x}}_{\boldsymbol{i}}\,. \tag{19}$$

Once the matrices X_i, X_i^{-1}, and D_i have been determined, a series of multiplications and additions of matrices and vectors is calculated at each time step.

3. RESULTS AND DISCUSSIONS

To implement the PCA method applied to modified point kinetics, a computer code called MCP-PCA was developed using Wolfram Mathematica® 6.0 software and, specifically, the intrinsic functions "Eigenvalues" and "Eigenvectors" were used to calculate the eigenvalues and eigenvectors of the M_i matrix. The simulations were compared with a reference model obtained using the finite difference method applied to modified point kinetics, MCP-FD, developed by Nunes, 2015. To generate the results, two types of reactivity insertion were considered: case 1, step-type reactivity insertion at three levels (300, 700, and 800pcm), and case 2, ramp-type reactivity insertion. The nuclear data used were the kinetic parameters from [26], shown in Table 1, and the other parameters for six groups of neutron precursors, shown in Table 2, widely used in the literature for the fissile isotope U-235, e.g., [11] and [20], among others. For comparison purposes, the simulations were carried out with two values of neutron transport frequencies $1/\tau$ equal to $10^3\ s^{-1}$ and $10^4\ s^{-1}$ $(\tau = 10^{-3}s,\ 10^{-4}s)$. According to [26], taking into account the available experimental results, the range of values for the neutron transport frequency varies from $10^3\ s^{-1}$ to $10^6\ s^{-1}$, with more severe transients implying lower values for this parameter. For each simulation, the execution time (CPU time) using the Mathematica® software was not precisely timed, but we can say that it was around 1 minute.

3.1. Case 1 - Step-type reactivity insertion

Three reactivity insertions were considered: prompt-subcritical insertion of $300\ pcm$, prompt-critical insertion of $700\ pcm$, and prompt-supercritical insertion of $800\ pcm$. The results obtained are shown in Table 3. The time steps, Δt, used in the MPC-PCA method were $0.1\ s$ for the $300\ pcm$ insertion and $0.01\ s$ for the $700\ pcm$ and $800\ pcm$ insertions. This table lists the results obtained using the finite difference method, MPC-FD [26], with a $10^{-6}s$ time step. The absolute relative percentage deviations between the MPC-PCA and MPC-FD methods were also calculated using the equation

$$PRD = \left| \frac{n(t)_{MPC-PCA} - n(t)_{MPC-FD}}{n(t)_{MPC-FD}} \right| \times 100\% \,. \tag{21}$$

The deviations calculated using Eq. 21 are shown in Table 4.

Table 1: Parameters of the modified kinetics for six groups of precursors.

Parameters	Value
Λ	0.00002 s
$\beta = \beta_1 + \beta_2 + \cdots + \beta_6$	0.007
Σ_a	0.1718 cm^{-1}
D	10 cm
v	3×10^5 cm/s
f_a	51,540 s^{-1}
τ	10^{-3} s, 10^{-4} s

Table 2: Precursor group parameters for U-235.

Precursors group	1	2	3	4	5	6
$\lambda_i(s^{-1})$	0.0127	0.0317	0.115	0.311	1.40	3.87
β_i	0.000266	0.001491	0.001316	0.002849	0.000896	0.000182

For the insertion of prompt-subcritical reactivity of 300 pcm, there is no divergence between the MPC-PCA and MPC-FD methods, both for the inverse of the transport frequency, relaxation time, for the insertion of prompt-subcritical reactivity of $300\,pcm$, there is no divergence between the MPC-PCA and MPC-FD methods, both for the inverse of the

Table 3: Neutron population obtained by the MPC-PCA and MPC-FD methods.

$\rho = 300\ pcm$					
Method	$\Delta t(s)$	$\tau(s)$	$t = 1.0\ s$	$t = 10\ s$	$t = 20\ s$
MPC-PCA	0.1	10^{-4}	2.20888	8.01333	2.826540
MPC-PCA	0.1	10^{-3}	2.20012	7.96040	2.797890
MPC-FD	10^{-6}	10^{-4}	2.20889	8.01328	2.826500
MPC-FD	10^{-6}	10^{-3}	2.20039	7.96101	2.798070
$\rho = 700\ pcm$					
Method	Δt (s)	τ (s)	$t = 0.01\ s$	$t = 0.5\ s$	$t = 2.0\ s$
MPC-PCA	0.01	10^{-4}	3.94994	2.97514×10^{3}	2.49744×10^{10}
MPC-PCA	0.01	10^{-3}	2.21233	2.70710×10^{2}	5.62973×10^{6}
MPC-FD	10^{-6}	10^{-4}	3.95463	2.97481×10^{3}	2.49504×10^{10}
MPC-FD	10^{-6}	10^{-3}	2.29174	2.71857×10^{2}	5.65158×10^{6}
$\rho = 800\ pcm$					
Method	Δt (s)	τ (s)	$t = 0.01\ s$	$t = 0.1\ s$	$t = 1.0\ s$
MPC-PCA	0.01	10^{-4}	5.17307	6.12178×10^{2}	1.77838×10^{20}
MPC-PCA	0.01	10^{-3}	2.50318	4.07905×10^{1}	2.25472×10^{9}
MPC-FD	10^{-6}	10^{-4}	5.18009	6.11999×10^{2}	1.76198×10^{20}
MPC-FD	10^{-6}	10^{-3}	2.60955	4.13621×10^{1}	2.27915×10^{9}

transport frequency, relaxation time, of $10^{-3}s$ and for $10^{-4}s$, where the maximum PRD is 0.01227%. With the insertion of prompt-critical reactivity of $700\ pcm$, a PRD of 3.46% is observed at the initial instant of the transient, $t = 0.01\ s$, with $\tau = 10^{-3}s$. This is not the case with $\tau = 10^{-4}s$, where a PRD of 0.11% was observed. Finally, there is the insertion of prompt-supercritical reactivity $800\ pcm$. Similar behavior is observed in this situation, which is even more extreme than the previous one, with a large PRD (4.07618%) at the initial instant of the transient, t=0.01 s, with $\tau = 10^{-3}s$. For the case of $\tau = 10^{-4}s$, the maximum PRD observed (0.93077%) occurs at the end of the transient. The parameter τ represents how much the modified point kinetics model diverges from the classical point kinetics model. However, this parameter also has an influence when comparing the results of two different

numerical methods, MPC-PCA and MPC-FD, to implement the modified punctual kinetics model because, from the analysis in Table 4, regardless of the transient, the type of reactivity insertion, there are more significant relative deviations when $\tau = 10^{-3}s$ compared to the deviations for $\tau = 10^{-4}s$.

Table 4: Absolute relative percentage deviation between the MPC-PCA and MPC-FD methods.

$\rho = 300\ pcm$			
τ (s)	$t = 1.0\ s$	$t = 10\ s$	$t = 20\ s$
10^{-4}	0.00045	0.00062	0.00142
10^{-3}	0.01227	0.00766	0.00643
$\rho = 700\ pcm$			
τ (s)	$t = 0.01\ s$	$t = 0.5\ s$	$t = 2.0\ s$
10^{-4}	0.11860	0.01109	0.09619
10^{-3}	3.46505	0.42191	0.38661
$\rho = 800\ pcm$			
τ (s)	$t = 0.01\ s$	$t = 0.1\ s$	$t = 1.0\ s$
10^{-4}	0.13552	0.02925	0.93077
10^{-3}	4.07618	1.38194	1.07189

3.2. Case 1 - Ramp-type reactivity insertion

In this case, a ramp-type reactivity insertion transient was numerically simulated, with a speed of \$0.1/s, as proposed by [11]:

$$\rho(t) = \$0.1t = 0.0007t$$

The kinetic parameters and those of the six precursor groups are identical to those used in the transients with step reactivity described in case 1. The simulations are carried out with time steps of 0.001 and 0.0001 s to test the sensitivity of the response to this variation. The results obtained using the MPC-PCA method are compared with those obtained using the MPC-FD method presented by [30]. The solution of classical point kinetics, calculated using Henry's θ-weighting method, PC-θ, with Δt = 0.0001s [11], is also considered for

comparison. The variations in the number of neutrons obtained by the methods for the case of Δt = 0.001s are shown in Table 5, while for Δt = 0.0001s, they are shown in Table 6. The relative percentage deviations are listed in Table 7, where the values of the time steps and the parameter τ are considered.

Table 5: Neutron population obtained by the MPC-PCA and MPC-FD methods and PC-θ. Ramp-type reactivity and step Δt = 0.001s.

Time	MPC-PCA		MPC-FD		PC-θ
	$\tau = 10^{-3}$s	$\tau = 10^{-4}$s	$\tau = 10^{-3}$s	$\tau = 10^{-4}$s	
t = 2s	1.33596	1.33809	1.33583	1.33796	1.3382
t = 4s	2.21820	2.22774	2.21783	2.22737	2.2283
t = 6s	5.49652	5.57486	5.49481	5.57311	5.5815
t = 8s	3.90654×10^{1}	4.24043×10^{1}	3.90352×10^{1}	4.23697×10^{1}	4.2781×10^{1}
t = 9s	3.40437×10^{2}	4.67905×10^{2}	3.39924×10^{2}	4.67082×10^{2}	4.8745×10^{2}
t = 10s	3.73409×10^{4}	2.98746×10^{5}	3.72105×10^{4}	2.97080×10^{5}	4.5109×10^{5}
t = 11s	1.93155×10^{9}	5.00610×10^{14}	1.91712×10^{9}	4.92958×10^{14}	1.7919×10^{16}

Table 6: Neutron population obtained by the MPC-PCA and MPC-FD methods and PC-θ. Ramp-type reactivity and step Δt = 0.0001s.

Time	MPC-PCA		MPC-FD		PC-θ
	$\tau = 10^{-3}$s	$\tau = 10^{-4}$s	$\tau = 10^{-3}$s	$\tau = 10^{-4}$s	
t = 2s	1.33585	1.33798	1.33583	1.33796	1.3382
t = 4s	2.21787	2.22741	2.21783	2.22737	2.2283
t = 6s	5.49498	5.57329	5.49481	5.57311	5.5815
t = 8s	3.90382×10^{1}	4.23731×10^{1}	3.90352×10^{1}	4.236967×10^{1}	4.2781×10^{1}
t = 9s	3.39975×10^{2}	4.67164×10^{2}	3.39923×10^{2}	4.670813×10^{2}	4.8745×10^{2}
t = 10s	3.72230×10^{4}	2.97236×10^{5}	3.72100×10^{4}	2.97068×10^{5}	4.5109×10^{5}
t = 11s	1.91829×10^{9}	4.93204×10^{14}	1.91682×10^{9}	4.92394×10^{14}	1.7919×10^{16}

It can be seen from the analysis of the results in Table 7 that the relative percentage deviations calculated, even with the more significant step of Δt = 0.001s, were low at the start of the transient, growing progressively with the increase in reactivity inserted, up to maximums of 0.75% and 1.55% , at t=11s, for the modified point kinetics with $\tau = 10^{-3}$s

and $\tau = 10^{-4}$s respectively. It can be seen that up to 8s, the deviations are very close and that from then on, they are more significant for $\tau = 10^{-4}$s.

Table 7: Calculate the relative percentage deviation between the MPC-PCA and MPC-FD methods. Ramp-type reactivity, $\Delta t = 0.001$s and $\Delta t = 0.0001$s.

Time	$\Delta t = 0.001$s		$\Delta t = 0.0001$s	
	$\tau = 10^{-3}$s	$\tau = 10^{-4}$s	$\tau = 10^{-3}$s	$\tau = 10^{-4}$s
t = 2s	0.00973	0.00972	0.00150	0.00149
t = 4s	0.01668	0.01661	0.00180	0.00180
t = 6s	0.03112	0.03140	0.00309	0.00323
t = 8s	0.07737	0.08166	0.00774	0.00809
t = 9s	0.15092	0.17620	0.01530	0.01770
t = 10s	0.35044	0.56079	0.03494	0.05655
t = 11s	0.75269	1.55226	0.07669	0.16450

When the step is reduced to Δt = 0.0001s, the deviations are reduced by almost an order of magnitude, with maximums of 0.077% and 0.164% at t=11s for the modified point kinetics with $\tau = 10^{-3}$s and $\tau = 10^{-4}$s, respectively. The observed behavior suggests using small time steps for transients with large reactivity insertion. It can be seen that from instant 6s onwards, the deviations are more significant for $\tau = 10^{-4}$s.

Figs. 1 and 2 show the graphs for the neutron population as a function of time, comparing the classical point kinetics, PC-θ, with the MPC-PCA method with $\tau = 10^{-3}$s, $\tau = 10^{-4}$s and Δt = 0.0001s. For convenience of visualization, the two graphs are on different time scales.

Figure 1: Graph of the neutron populations of classical point kinetics, modified point kinetics with $\tau = 10^{-3}$s and $\tau = 10^{-4}$s . Ramp-type reactivity.

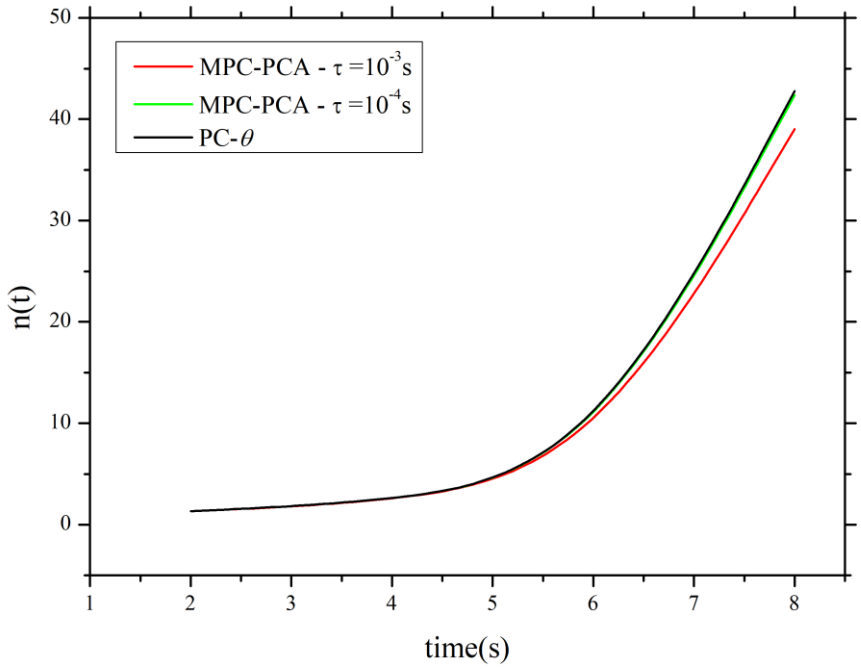
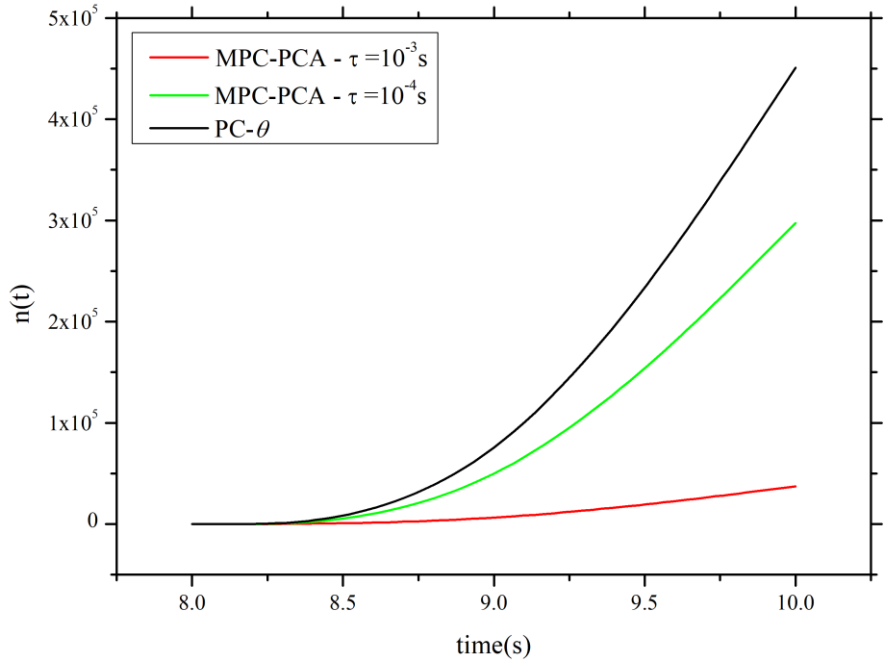


Figure 2: Graph of the neutron populations of classical point kinetics, modified point kinetics with $\tau = 10^{-3}$s and $\tau = 10^{-4}$s . Ramp-type reactivity.



It can be seen from the graph in Fig. 1 that the curves for classic point kinetics (black curve) and modified point kinetics with $\tau = 10^{-4}$s (green curve) practically overlap on the time scale up to 8s, while modified point kinetics with $\tau = 10^{-3}$s (red curve) begins to move away from the classic point kinetics curve. Considering the graph in Fig. 2, with a time scale starting at 8s, the difference between the classical point kinetics and the modified point kinetics models can be seen when the reactivity increases to the point where it gets close to the fraction of delayed neutrons $\beta = 0.007$, and the most significant divergence can be seen for the modified point kinetics with $\tau = 10^{-4}$s. This behavior was expected because as the value of the relaxation time τ decreases, the closer the modified point kinetics gets to classical point kinetics.

4. CONCLUSIONS

In this work, we set out to apply the PCA method, originally used to solve the classical point kinetics equations, to obtain the solution of the modified point kinetics equations for six groups of precursors. The modified point kinetics model can be derived from the time-dependent neutron transport equations and consists of not neglecting the time derivative for the neutron current density in the equations that lead to the neutron diffusion model. An important parameter that arises with the formulation of the modified point kinetics is the relaxation time. If this parameter is null, the modified point kinetics returns to classical point kinetics. The PCA method approximates the source and reactivity functions by piecewise continuous functions, transforming the problem into a system of linear ordinary differential equations that can be solved exactly for each time step. The modified point kinetics equations were rewritten conveniently to apply the PCA method. A system of approximate differential equations is obtained, and the matrix is diagonalized. Simple matrix and vector operations are performed at each time step. The results obtained were compared with a reference, given by the finite difference method applied to modified point kinetics to verify the proposed application. Two types of transients were considered: step-type reactivity insertion and ramp-

type reactivity insertion. In the first case, three levels of insertions were tested: prompt-subcritical, prompt-critical, and prompt-supercritical. For the prompt-subcritical insertion, the PCA method showed similar results to the finite difference method, regardless of the relaxation time value. In the case of the prompt-critical and prompt-supercritical reactivity insertions, the results obtained were similar between the two methods only for the shorter relaxation time since for the higher value of this parameter, the results differed between the methods, especially in the initial instant of the transient. Both methods showed similar results in the initial transient moments in the ramp-type reactivity insertion transient. However, from a specific moment onwards, the results began to diverge, with the deviation increasing more with time for the case with the shorter relaxation time compared to the case with the longer relaxation time. In this transient, we also noticed that the PCA method for the modified kinetics presents a first-order truncation error in the temporal discretization. Finally, we also compared the proposed method with the classical point kinetics method. We found that the proposed method showed consistent behavior when the smallest value of the relaxation time is considered.

We suggest that the methodology proposed in this work be tested on other types of more complex transients than those used in cases 1 and 2. We believe this methodology can be improved if we consider a complementary method for optimizing the relaxation time value. In addition, studies are recommended to verify the possibility of treating the discretization in time using an automatic control on the time step size.

CONFLICT OF INTEREST

All authors declare that they have no conflicts of interest.

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