doi.org/10.15392/2319-0612.2025.2963
Braz. J. Radiat. Sci., Rio de Janeiro
2025, 13(4) | 01-17 | e2963
Editor: Prof. Dr. Bernardo Maranhão Dantas
Editor: Prof. Dr. Alfredo Lopes Ferreira Filho
Submitted: 2025-06-05
Accepted: 2025-09-22

Evaluation of Nuclear Material Balance by Statistical Analysis of the Material Unaccounted For (MUF)

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Abstract: Evaluating the nuclear material balance is an essential tool that can be used to assess the validity of the operator's declarations of existing quantities of nuclear material from a facility, with an effect on safeguards agreements, where applicable. In summary, evaluating the nuclear material balance and associated safeguards activities enables the national regulatory authority to identify variations that may indicate deviations. A specific strategy used to detect potential deviations from assessing the material balance is known as "diversion into MUF", where MUF refers to the parameter Material Unaccounted For. It represents the difference between the book inventory and the ending physical inventory at the end of the material balance period. This evaluation is conducted through statistical analysis of the standard deviation of the MUF (σMUF) parameter, using an anonymous Brazilian facility as a case study. The ultimate goal is to implement this analysis within Brazil's control and accounting system, known as e-Gamma. This will enable the system to automatically perform such analyses for all facilities nationwide during each accounting period. This statistical study can be a powerful predictor of diversion, utilizing the σ_{MUF} parameter as a valuable resource. It is possible to suggest implementing this evaluation in control and accounting systems to increase efficiency in identifying and resolving discrepancies and anomalies, which is of fundamental importance in improving the reliability of a control system within the national scenario. Considering the international scenario, a country that increases its capacity to verify the accounting of nuclear material under its custody will tend to occupy a position of autonomy among its peers.

Keywords: nuclear material balance; safeguards; diversion of nuclear material; material unaccounted for

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Avaliação do Balanço de Material Nuclear por Análise Estatística de Material Não Contabilizado

Resumo: A avaliação do balanço de material nuclear é uma ferramenta essencial utilizada para avaliar a validade das declarações do operador quanto às quantidades existentes de material nuclear de uma instalação e para verificar o cumprimento dos acordos de salvaguardas, quando existentes. Em resumo, a avaliação do balanço de material e as atividades de salvaguardas associadas a esta fornecem à autoridade regulatória nacional a capacidade de detectar desvios de material nuclear. Uma estratégia específica utilizada para detectar potenciais desvios na avaliação do balanço de materiais é conhecida como "desvio para o MUF", onde MUF se refere ao parâmetro Material Não Contabilizado. MUF é a diferença entre o inventário de livro e o inventário físico final ao fim do período de balanço de materiais. Essa avaliação é realizada por meio de análise estatística utilizando o desvio padrão do parâmetro MUF (σ_{MUF}), utilizando uma instalação brasileira anônima como estudo de caso. O objetivo final e valioso é a implementação desta análise no sistema brasileiro de controle e contabilidade, chamado e-Gamma. Assim, esta análise poderá ser realizada automaticamente pelo sistema para todas as instalações do país, por período contábil. Este estudo estatístico pode ser o preditor mais poderoso de desvio, utilizando o parâmetro σ_{MUF} como um recurso valioso. É possível implementar essa avaliação em sistemas de controle e contabilidade para aumentar a eficiência na identificação e resolução de discrepâncias/anomalias, o que é de fundamental importância para melhorar a confiabilidade de um sistema de controle em termos do cenário nacional. E tendo em vista o cenário internacional, um país que aumenta sua capacidade de verificação da contabilidade do material nuclear sob sua custódia tenderá a ocupar uma posição de autonomia no contexto de seus pares.

Palavras-chave: balanço de material nuclear, salvaguardas, desvio de material nuclear, material não contabilizado

1. INTRODUCTION

Accounting practices applied to nuclear materials are fundamental to nuclear material control. This accounting process relies on the nuclear material balance. This mass balance involves measuring inventories and material flow both within and outside the facility, as well as within its confines. Maintaining this balance enables the determination of the quantities of nuclear material present in each location and the detection of any quantitative discrepancies. The emphasis is placed on evaluating the balance of uranium, specifically its isotope U-235, along with plutonium. Control measures are implemented through Material Balance Areas (MBAs), which are considered essential accounting units for recording and reporting to regulatory authorities and safeguard agencies, as applicable. Any material that crosses the facility's boundaries (either as input or output) must be documented as an inventory change. In contrast, the material contained within must be reported as part of the physical inventory. In this context, physical inventory refers to the total of all quantities measured or estimated based on prior assessments of each batch of nuclear material present at a specific time within the material balance area of the nuclear facility [4, 5]

The material balance period is between two consecutive Physical Inventory Taking (PIT) events, and the material balance evaluation is conducted annually following the PIT for each MBA and category of nuclear material [3].

PIT serves as a critical focal point for nuclear material accounting activities. By evaluating the material balance concerning the PIT and assessing the facility's inputs and outputs, it is possible to gather definitive evidence of the presence of nuclear material and identify potential significant losses. During the PIT, the results of the physical inventory are also compared with the values contained in the records (book inventory), and any significant differences resulting from both are investigated and resolved.

The purpose of evaluating the Material Unaccounted For (MUF) in a material balance is to determine whether it can be plausibly explained (measurement errors, process losses, etc.), or whether there is any indication of a possible diversion of nuclear material that could be hiding in the MUF, without falsification of accounting records.

Finding a MUF value different than zero may occur for many reasons, such as normal errors (random and systematic) inherent to all measurements, abnormal errors due to malfunctioning of measuring instruments, unmeasurable losses of nuclear material, gross measurement errors, material retained in the process and not measured and diversion of nuclear material [2].

Suppose the MUF is caused solely by measurement errors. In that case, it is possible to determine the standard deviation (σ_{MUF}) of the MUF using an error propagation model, based on the standard deviation of the operator's measurement methods. If a MUF value is obtained in a given period that is far from the mean value (zero), it may be indicative of an anomaly. In other words, if the MUF value under the influence of measurement errors is excluded, the real MUF value is obtained, which in an ideal situation should be zero. In practice, however, the real MUF value is not zero due to the reasons given above. Therefore, it is essential to calculate σ_{MUF} to evaluate the MUF.

This technical note explores the significance of Material Unaccounted For in the context of safeguards and the detection of material deviations. This study investigates the uncertainty inherent in the Material Unaccounted For (MUF) value, as quantified by its standard deviation, σMUF. The analysis is conducted within the context of an anonymous Brazilian facility to maintain the confidentiality of sensitive information. The primary objective of this research is to facilitate the integration of this analytical framework into Brazil's control and accounting system, known as e-Gamma. By implementing this methodology, the system will have the capability to automatically perform this analysis for all facilities across the nation on a per-accounting period basis.

The e-Gamma system is Brazil's control and accounting platform, available at www.e-gamma.cnen.gov.br. It maintains a comprehensive inventory of all nuclear facilities, monitors changes in that inventory, and produces accounting reports for all facilities nationwide. This system impacts all levels of the nuclear sector, from regulatory agencies to nuclear facilities, including those operated by the Armed Forces, the Brazil-Argentina Agency for Accounting and Control of Nuclear Materials (ABACC), and the International Atomic Energy Agency (IAEA).

Evaluating the MUF is a crucial aspect of nuclear material control, as it facilitates the identification of deviations that measurement errors may hide. By conducting this straightforward statistical study and integrating it into e-Gammait is ensured that the national regulatory authority, the National Atomic Energy Commission (CNEN), will enhance its effectiveness in identifying and addressing potential discrepancies and anomalies. This improvement is expected to positively influence the reliability of the national control system. It may also elevate Brazil's standing on the international stage, fostering a sense of autonomy and trust. The approach encompasses various areas of interest, including nuclear safety, the operation of nuclear facilities, physical security, and nuclear safeguards. On an international scale, this initiative holds significant value in detecting deviations in nuclear material that could be relevant to the production of nuclear devices. Ultimately, this effort may help mitigate potential conflicts with international standards aimed at preventing the proliferation of weapons of mass destruction and advancing the country's technological capabilities.

2. MATERIALS AND METHODS

The Material Unaccounted For is defined as the difference between the book inventory (expected value) and the physical inventory (found value) at the time of the PIT at the end of the material balance period, as shown in equation 1.

$$\text{MUF} = \text{BA} - \text{PE} \qquad (1)$$

where *BA* is the book inventory at the end of the material balance, and *PE* is the physical inventory at the end of the period.

The sum of the beginning physical inventory and all changes that have occurred since that inventory was taken is the quantity of material on the facility books; therefore, MUF can also be described as presented by Equation 2.

$$\text{MUF} = (\text{PB} + \text{X} - \text{Y}) - \text{PE} \qquad (2)$$

where *PB* is the Physical Beginning inventory, *X* is the sum of inventory changes that represent additions (entries) to inventory during the period, *Y* is the sum of inventory changes that represent decreases (outflows) to inventory during the period, and *PE* is the Physical Ending inventory.

To calculate σ_{MUF}, it is necessary first to calculate the variance of each of the four components of the MUF equation. Therefore, the σ_{MUF} equation can be expressed as shown in equation 3.

$$\boldsymbol{\sigma^2_{MUF} = \sigma^2_{PB} + \sigma^2_{X} \mp \sigma^2_{Y} \mp \sigma^2_{PE}} \qquad (3)$$

where σ^2_{MUF} is the variance of the MUF, σ^2_{PB} is the variance of the beginning physical inventory, σ^2_{X} is the variance of the inventory additions of the period, σ^2_{Y} is the variance of the inventory decreases, and σ^2_{PE} is the variance of the ending physical inventory.

Combining the variances and taking the square root gives the value of σ_{MUF}. To calculate the four variances, it is necessary to compile data for each of the four components, including the amount of each form (e.g., oxide, powder, pellets) of nuclear material that was measured during the inventory period. The uncertainty associated with each measurement method used will assign values to each form of nuclear material (e.g., non-destructive analysis, chemical sampling, mass volume, etc.) and the number of measurements made. This process is referred to as error propagation (or propagation of measurement uncertainty) [1] as shown in equation 4.

$$\sigma_{PE}^2 = M_{PE}^2 \left\{ \frac{(\delta_{PE,r})^2}{n_{PE}} + \left(\delta_{PE,s}\right)^2 \right\} \quad (4)$$

where M_{PE} is the quantity of material in the final physical inventory, $\delta_{PE,r}$ is the random uncertainty of the measurement, n_{PE} is the number of measurements performed, and $\delta_{PE,s}$ is the systematic uncertainty of the measurement.

The same methodology must be applied to the four components of the MUF equation to obtain the value of σ_{MUF}. As mentioned previously, measurement uncertainty is expressed as a percentage relative standard deviation (δ). These uncertainties are specified based on the MBA, material type, stratum, and, in the case of MUF calculation, it will also be essential to analyze the measurement system (method and instrument). Suppose an MBA measures plutonium powder using non-destructive analysis (NDA) and also measures it using chemical analysis (CA). In that case, two different sets of measurement uncertainty estimates will be associated with the two other systems used to measure the material. Suppose an MBA has both homogeneous and heterogeneous powder. In that case, two different sets of measurement uncertainty estimates will be needed to describe the measurement characteristics associated with the two distinct forms of material.

International standards for measurement uncertainty are outlined by the International Atomic Energy Agency, referred to as "Target Values" [6]. Furthermore, when calculating the variance of the Unaccounted Material For and its standard deviation, it is vital to confirm that this value falls within the confidence limits. The IAEA employs a 3σ decision rule, established by the Technical Review Committee [2]. The 3σ value corresponds to a probability of less than 0.3% for incorrectly identifying a problem when none exists. The benefit of using 3σ is that any alarm raised indicates a genuine issue, with the likelihood of a false alarm occurring fewer than 3 times out of 1000.

Another critical point to consider when analyzing MUF is static material. Material that was not measured during the balance period cannot contribute to MUF, since it is static. Since it does not contribute to MUF, it should be excluded from the calculation of σ_{MUF}. The consequence of not excluding static material is to overestimate σ_{MUF} and lead to the erroneous acceptance of a MUF that is significantly different from zero. Therefore, it is critical to exclude static material from the material balance table before the sigma MUF is calculated.

Static material is material that appears identically in two opposite components of the material balance equation, having remained unchanged throughout the material balance period. Dynamic material is a type of material that is not static, typically because it has undergone some form of processing or treatment. The two categories are mutually exclusive, i.e., material that is not static is dynamic and vice versa.

Assuming that the identification of the items is consistent, static material can be identified using the computer. Receipts and other increases, along with physical beginning, comprise one data set in the search comparison. In contrast, shipments and other decreases, along with physical ending, comprise the other data set. If an item appears in PB and increases with the same identification and the same amounts of elements/isotopes as in decreases and PE, then it is static.

3. RESULTS AND DISCUSSIONS

Data from the Brazilian Control and Accounting System, e-Gamma [7], was utilized to evaluate material balance based on the significance of the Unaccounted Material For (MUF). However, since this information is confidential, the nuclear facility will be referred to anonymously as BR-X, although all data used to illustrate this evaluation is accurate. Table 1 presents the material balance for an anonymous MBA BR-X covering the balance period from November 5, 2022, to July 30, 2023, where several points are listed as follows:

(a) The material balance is categorized by type, with D representing depleted, E for enriched, and N for natural material;

(b) The first column displays the start and end dates for the period when all materials are accounted for. ThePB was recorded on November 5, 2022 and the PE was determined during the PIT on July 30, 2023;

(c) The second column identifies the material category (D, E, and N);

(d) The third column outlines the types of inventory variations during the period, where DE is category change, RD is domestic receipt, RF is import, SD is domestic shipment, SF is export, LD is measured discard, and DI is shipment-receiver difference. In addition, there is the PE, the BA, which is based on everything that the operator declared in the inventory variations for the period, the PE – Physical Ending, which is obtained with the PIT on 07/30/2023, and the MUF which is the difference between the book inventory and the final physical inventory (BA - PE).

(e) The final two columns represent the total weight of uranium and its corresponding isotope, as applicable.

Table 1: Material Balance of hypothetical MBA BR-X.

MATERIAL INVENTORY				
MBA: BR-X		**QUERY PERIOD: 11/05/2022 – 07/30/2023**		
DATE	**CATEGORY**	**IC**	**ELEM.W (g)**	**ISOT.W (g)**
11/05/2022	D	PB	46,373,624	
		DE	-36,794	
07/30/2023		BA	46,336,830	
		PE	46,319,551	
		MF	17,279	
11/05/2022	E	PB	96,269,425	3,040,830
		DE	36,794	74
		RD	28,730	1,147
		RF	14,510,614	599,949
		SD	-28,193,426	-1,188,464
		SF	-3,001,088	-124,825
		LD	-60	-3
		DI	3,042	125
07/30/2023		BA	79,654,031	2,328,833
		PE	79,728,954	2,331,095
		MF	-74,923	-2,262
11/05/2022	N	PB	454,700	
07/30/2023		BA	454,700	
		PE	454,700	
		MUF	0	

For the analysis of unaccounted material, data already made available by e-Gamma will be used [7]. From Table 1, it is possible to observe in red the value of the MUF parameter for the period, where for the enriched uranium category, it is -74,923 g of the element U and -2,262 g of the isotope U-235.

The system has a function that will be essential for the analysis of this parameter: the automatic comparison of the Items List of the previous Physical Inventory Taking with the current PIT, indicating which batches have undergone changes, that is, identifying which material in that period is dynamic, and indicating if this batch was excluded, included or had its mass changed within that accounting period. This information is of utmost importance, as previously discussed in the Materials and Methods section. Material that was not measured during the balance period, known as *static*, cannot contribute to the MUF. Since it does not contribute to the MUF, it must be excluded from the calculation of σ_{MUF} to avoid an overestimation of the σ_{MUF}.

To determine the uncertainty of the material balance, σ_{MUF} must consider the variables such as the quantity of each form (e.g., oxide, powder, pellets, etc.), the uncertainty associated with each of the measurement methods used (e.g., non-destructive testing, sampling, chemical analysis, etc.), and the number of measurements made.

Thus, the first step in this evaluation is to build a table called the Material Balance Table (MBT). The MBT is a stratified list of all the material in each of the components (PB, X, Y, and PE) into distinct groups (strata). Each row must be assigned to one of the strata. The MBT is an MxN matrix (using the term "matrix" in the sense of a structured matrix), where the M rows of the matrix are the strata that constitute the stratum identification codes (type of material) and the identifier of the component of the material balance equation (initial physical inventory, increases, decreases and final physical inventory). The N columns represent the stratum variables, which include the weight of the element or isotope, the number of items (or batches), the number of measurements, and the uncertainty associated with each measurement method used. The Material Balance Table for this study was constructed and is available in Table 2, with:

(a) The first column displays the stratum by type of material and the identifier of the component of the material balance equation, beginning inventory (BI), increases (X), decreases (Y), and ending inventory (EI);

(b) The second and third columns present the element U and the isotope U-235 weight;

(c) The fourth and fifth columns show the number of batches and items for each stratum;

(d) And the last 10 columns are the random (δ_r) and systematic (δ_s) uncertainty associated with each of the measurement methods used based on the "Target Values" [6], where EBAL is the weighing method with electronic balance, DA (Destructive Analysis) is Sampling Uncertainties for U and U-235 Element Mass Fraction, gravimetry for Uranium Element Mass Fraction Measurements and ICPMS for U-235 Isotope Ratio Measurements.

Table 2: The Material Balance Table for the period.

Strata	U W. (g)	U235 W. (g)	Batch	Item	δr EBAL	δs EBAL	δr DA	δs DA	δr DA (U235)	δs DA (U235)	δr Grav.	δs Grav.	δr ICPMS	δs ICPMS
Fuel Element (X)	7,640,838	306,102	21	21	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.1	0.1
Gd Fuel Element (Y)	28,162,341	1,187,171	52	52	0.05	0.05	0.2	0	0.1	0.1	0.1	0.1	0.1	0.1
Gd Fuel Element (X)	7,503,303	299,926	20	20	0.05	0.05	0.2	0	0.1	0.1	0.1	0.1	0.1	0.1
Rod (Y)	9,612	404	2	5	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.1	0.1
Rod (X)	32,037	1,280	2	20	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.1	0.1
Gd Rod (X)	305,697	8,477	3	204	0.05	0.05	0.2	0	0.1	0.1	0.1	0.1	0.1	0.1
Gd Rod (BI/EI)	8,853	230	1	5	0.05	0.05	0.2	0	0.1	0.1	0.1	0.1	0.1	0.1
Powder (X)	456	19	1	1	0.05	0.05	1	0	0.05	0.05	0.05	0.05	0.1	0.1
Powder (X)	56,938	2,363	1	2	0.05	0.05	1	0	0.05	0.05	0.05	0.05	0.1	0.1
Powder (BI/EI)	8,660	360	1	2	0.05	0.05	1	0	0.05	0.05	0.05	0.05	0.1	0.1
UO2 Powder (Y)	3,174,628	132,387	6	7	0.05	0.05	0.2	0	0.05	0.05	0.05	0.05	0.1	0.1
UO2 Powder (X)	1,009,277	41,855	1	14	0.05	0.05	0.2	0	0.05	0.05	0.05	0.05	0.1	0.1
U3O8 Powder (Y)	472,117	19,558	1	28	0.05	0.05	0.2	0	0.05	0.05	0.05	0.05	0.1	0.1
U3O8 Powder (X)	826,557	32,803	2	14	0.05	0.05	0.2	0	0.05	0.05	0.05	0.05	0.1	0.1

Strata	U W. (g)	U235 W. (g)	Batch	Item	δr EBAL	δs EBAL	δr DA	δs DA	δr DA (U235)	δs DA (U235)	δr Grav.	δs Grav.	δr ICPMS	δs ICPMS
U3O8 Powder (BI/EI)	4,073,043	132,139	4	19	0.05	0.05	0.2	0	0.05	0.05	0.05	0.05	0.1	0.1
Pellets (Y)	1,988,136	63,772	7	79	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.1	0.1
Pellets (X)	425,855	17,554	2	13	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.1	0.1
Pellets (BI/EI)	1,741,537	67,673	3	50	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.1	0.1
Gd Pellets (Y)	254,207	7,224	8	8	0.05	0.05	0.2	0	0.1	0.1	0.1	0.1	0.1	0.1
UF6 Solution (X)	2,772	111	1	129	0.05	0.05	0.1	0	0.05	0.05	0.05	0.05	0.1	0.1
UF6 Solution (BI/EI)	11,170	464	3	13	0.05	0.05	0.1	0	0.05	0.05	0.05	0.05	0.1	0.1
UF6 Cilynder (Y)	30,831	1,279	6	6	0.05	0.05	0.05	0	0.05	0.05	0.05	0.05	0.1	0.1
UF6 Cilynder (X)	1,500,807	62,166	15	15	0.05	0.05	0.05	0	0.05	0.05	0.05	0.05	0.1	0.1
UF6 Cilynder (BI/EI)	35,022	1,402	26	26	0.05	0.05	0.05	0	0.05	0.05	0.05	0.05	0.1	0.1
Waste (Y)	153,218	5,647	9	9	0.05	0.05	10	0	0.05	0.05	0.05	0.05	0.1	0.1
Waste (X)	209,881	8,757	14	14	0.05	0.05	10	0	0.05	0.05	0.05	0.05	0.1	0.1
Waste (BI/EI)	63,543	2,641	4	4	0.05	0.05	10	0	0.05	0.05	0.05	0.05	0.1	0.1
UO2 Scrap (X)	1,190	31	1	1	0.05	0.05	1	0	0.05	0.05	0.05	0.05	0.1	0.1
UO2 Scrap (BI/EI)	96,745	4,074	2	2	0.05	0.05	1	0	0.05	0.05	0.05	0.05	0.1	0.1
UO2 Scrap (Y)	354,437	13,481	15	15	0.05	0.05	1	0	0.05	0.05	0.05	0.05	0.1	0.1
UO2 Scrap (X)	403,141	17,141	9	10	0.05	0.05	1	0	0.05	0.05	0.05	0.05	0.1	0.1
UO2 Scrap (BI/EI)	106,666	3,774	5	5	0.05	0.05	1	0	0.05	0.05	0.05	0.05	0.1	0.1
U3O8 Scrap (Y)	783,148	32,998	23	23	0.05	0.05	1	0	0.05	0.05	0.05	0.05	0.1	0.1
U3O8 Scrap (X)	446,142	18,514	11	11	0.05	0.05	1	0	0.05	0.05	0.05	0.05	0.1	0.1
U3O8 Scrap (BI/EI)	71,358	2,961	2	2	0.05	0.05	1	0	0.05	0.05	0.05	0.05	0.1	0.1
Gd Scrap (X)	5,966	170	1	1	0.05	0.05	1	0	0.1	0.1	0.1	0.1	0.1	0.1
Standard (X)	57,073	2,184	3	3	0.05	0.05	1	0	0.05	0.05	0.05	0.05	0.1	0.1

Strata	U W. (g)	U235 W. (g)	Batch	Item	δr EBAL	δs EBAL	δr DA	δs DA	δr DA (U235)	δs DA (U235)	δr Grav.	δs Grav.	δr ICPMS	δs ICPMS
Standard (BI/EI)	66	1	2	10	0.05	0.05	1	0	0.05	0.05	0.05	0.05	0.1	0.1
Sample (Y)	1,639	60	5	91	0.05	0.05	1	0	0.05	0.05	0.05	0.05	0.1	0.1
Sample (X)	6,158	244	7	61	0.05	0.05	1	0	0.05	0.05	0.05	0.05	0.1	0.1
Sample (BI/EI)	3,265	110	7	30	0.05	0.05	1	0	0.05	0.05	0.05	0.05	0.1	0.1

Using Equation 4, as presented in the Materials and Methods, and with the data available and organized in the MBT shown in Table 2, calculations were performed to determine the total variance for each measurement method for the element uranium (U) and its isotope U-235, as presented in Table 3. The difference in the calculation for U and U-235 is that for the element, the mass of U is used (column 2 of table 2), and for the isotope, the mass of U-235 is used (column 3 of table 2). The uncertainty values for the destructive analysis (DA) measurements vary. For the element, the values from columns 8 and 9 are used, and for the isotope, the values from columns 10 and 11 are used.

Table 3: Total variance for each measurement method.

Total variance for U							
$(\sigma_r)^2$ **EBAL**	$(\sigma_s)^2$ **EBAL**	$(\sigma_r)^2$ **DA**	$(\sigma_s)^2$ **DA**	$(\sigma_r)^2$ **Grav.**	$(\sigma_s)^2$ **Grav.**	$(\sigma_r)^2$ **ICPMS**	$(\sigma_s)^2$ **ICPMS**
7,551,533.06	236,794,410.20	179,373,653.42	16,387,628.12	21,131,466.78	873,975,804.45	30,206,132.22	947,177,640.78
Total variance for U-235							
$(\sigma_r)^2$ **EBAL**	$(\sigma_s)^2$ **EBAL**	$(\sigma_r)^2$ **DA**	$(\sigma_s)^2$ **DA**	$(\sigma_r)^2$ **Grav.**	$(\sigma_s)^2$ **Grav.**	$(\sigma_r)^2$ **ICPMS**	$(\sigma_s)^2$ **ICPMS**
12,182.40	411,562.09	134,603.10	25,686.48	35,906.17	1,536,153.13	48,729.59	1,646,248.37

Adding the variances found for all measurement methods, we obtain the total variance of the MUF with a value of 2,312,598,269.02 g of U and a MUF standard deviation (σ_{MUF}) of 48,089.48 g of U, which yields a confidence limit value of 3xσ_{MUF}, equal to 144,268.45 g. Taking into account that the MUF value for the period is 74,923 g of U, it is possible to confirm that this is within the confidence limit and is not considered statistically significant.

The same evaluation was performed for the element, adding the variances found for all measurement methods, we obtain the total variance of the MUF with a value equal to 3,851,071.33 g of U-235 and MUF standard deviation (σ_{MUF}) equal to 1,962.41 g U-235, thus obtaining the confidence limit value of 3xσ_{MUF} equal to 5,887.24 g. Taking into account that the MUF value for the period is 2,262 g of U-235, it is possible to confirm that this is within the confidence limit and is not considered statistically significant.

4. CONCLUSIONS

The evaluation of the material balance, conducted through statistical analysis of the unaccounted material for using the MUF standard deviation parameter (σ_{MUF}), indicates a safe value for the accounting period from November 5, 2022, to July 30, 2023. This finding allows us to assert that there is no evidence of nuclear material diversion during this period.

The study of this evaluation is of utmost importance for the accounting and control of nuclear material, presenting several notable advantages, particularly in increasing the efficiency in identifying and resolving discrepancies and anomalies. This improvement is essential to reinforce the reliability of control systems, thus increasing their autonomy and competitiveness in the international scenario.

ACKNOWLEDGMENT

The authors thank our colleagues from IME, CNEN, and ABACC who provided insight and expertise that greatly assisted the research.

FUNDING

The Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq) supported this work (CNPq Grants n° 304636/2023-1 and 409490/2023-7).

CONFLICT OF INTEREST

The authors declare no conflicts of interest in this article.

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