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DEVELOPED LIQUID FILMS FALLING AROUND TAYLOR BUBBLES INSIDE VERTICAL STAGNANT COLUMNS

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ABSTRACT

The present work reports an experimental study of developed liquid films falling around single Taylor bubbles inside vertical tubes containing stagnant liquids. Experiments were carried out in acrylic tubes with 2.0 m length and inner diameters of 0.019, 0.024 and 0.034 m. Five water-glycerin mixtures were used, corresponding to film Reynolds number (Re_f) ranging from 2 to 7650. A pulse-echo ultrasonic technique was applied to measure the rise velocity of the bubble and the equilibrium thickness of the liquid film. These parameters together with the calculated standard deviation of the equilibrium film thickness provided information about the development of waves on the gas-liquid interfaces, which could be related with the laminar-turbulent transition of liquid films falling around Taylor bubbles. The results indicated that the wave amplitudes increased sharply for $Re_f \geq 1000$. This value of Re_f is in agreement with literature concerning the laminar-turbulent transition for free falling films on vertical surfaces.

Keywords: **Taylor bubbles, Vertical tubes, Ultrasonic technique, Falling film, laminar-turbulent transition.**

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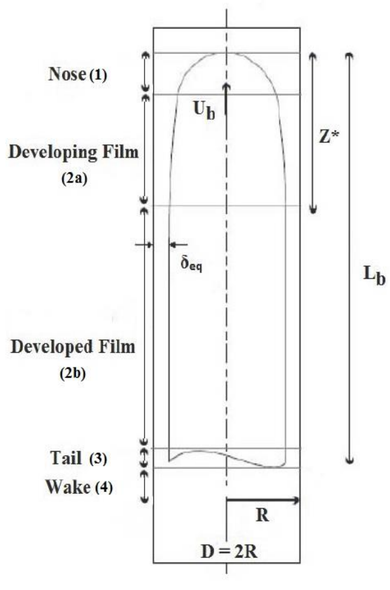
1. INTRODUCTION

Slug flow is an important and complex form of gas-liquid two-phase flows and is encountered in a wide range of industrial applications such as nuclear reactor cooling systems, pipeline systems for the transport of oil-gas mixtures, evaporators, boilers, condensers, among others. This flow pattern is characterized by long bullet-shaped bubbles, also called Taylor bubbles, and a liquid slug between successive bubbles. Fig.1 presents a schematic of a Taylor bubble flowing in a liquid. A typical Taylor bubble can be divided into four regions: (1) an approximately hemispherical nose, (2) a body region surrounded by a falling liquid film, (3) a tail region, and (4) a wake region. The body region can be further subdivided: (2a) around the upper part, where the developing film is accelerating and thinning, and (2b) around the lower part, where the forces acting on the film are in equilibrium and the film has constant velocity profile and thickness δ_{eq}. In Fig.1, Z* is the distance from the top of the bubble nose to where the film reaches the equilibrium, which is called film development length.

Figure1: *Schematic of a Taylor bubble flowing in a liquid.*



In nuclear reactors, the two-phase flow parameters need to be constantly controlled at the primary cooling system or during an emergency core cooling. Slug flow can be potentially hazardous to the pipe system structure or to the equipment and processes on nuclear reactors due to the strongly oscillating pressure levels formed behind liquid slugs. Additionally, slug flow is usually accompanied by fluctuations in pipe temperature, which can lead to high pipe wall temperatures resulting in "dryout" and causing damages in the nuclear power generating systems. Since the characteristics of two-phase gas-liquid slug flow can also influence mass and heat transfer processes, a thorough understanding of this flow pattern is fundamental for safe operation of nuclear reactors.

Taylor bubbles and slug flow have been studied for decades. The pioneering works of Dumitrescu [1] and Davies and Taylor [2] have studied, theoretically and experimentally, the propagation of infinite elongated bubbles in vertical tubes with emphasis on the propagation velocity and shape of the bubbles. Thenceforth many studies have been conducted to a better understanding of slug flow characteristics.

Morgado *et al*. [3] have presented a literature review of vertical gas-liquid slug flow, from 1943 to 2015, covering theoretical, experimental, and numerical approaches. Data concerning hydrodynamics of the liquid film surrounding Taylor bubbles proved scarce in the literature, leading these authors to conclude that future works on slug flow should also focus on this theme, as well as on experimental studies that ensure a clear understanding of the transition region in the film.

It is common to consider that when the film around an elongated bubble is thin, the local curvature of the tube can be neglected and the behavior of the liquid film is the same of a liquid falling down the inner or outer surface of a cylindrical tube or down a plane surface (Goldsmith and Mason [4], Brown [5]), which makes the literature of these topics relevant for the study of falling films around Taylor bubbles.

Liquid films are usually described in the literature by the film Reynolds number Re_f (Dukler and Bergelin [6]), defined as:

$$Ref = 4\,\frac{\rho L * vf * \delta}{\mu L}, \tag{1}$$

where μ_L is the liquid dynamic viscosity, ρ_L the liquid density, δ the film thickness, and v_f the film velocity.

According Fulford [7], it is well known in film flow that below a certain critical value of Re_f the flow will be mainly laminar, while above this value, turbulence becomes increasingly important. However, it must be considered that in thin films a large part of the total film thickness continues to be occupied by the relatively nonturbulent ``laminar sublayer", even at large flow rates. Hence, the transition from laminar to turbulent flow cannot be expected to be sharply marked. Nevertheless, it is of value to subdivide film flow into laminar and turbulent regimes depending on whether Re_f is smaller or higher than a critical value. This author consider that the laminar-turbulent transition occurs in the range of $1000 \leq Re_f \leq 2000$.

Studying falling liquid films on surfaces, other authors have proposed different ranges for the transition from laminar to turbulent flow regime. Dukler and Bergelin [6] have considered that the transition occurs for $Re_f \approx 1000$. The results obtained by Salazar and Marschall [8] indicated a laminar-turbulent transition zone for Re_f ranging from 600 to 1000. Aragaki *et al*. [9], by taking experimental and theoretical results into account, have deduced that the transition to a turbulent flow occurred in a definite range of Re_f, i.e. $150 \leq Re_f \leq 1000$.

Karapantsios *et al* [10] have studied experimentally the characteristics of liquid films flowing inside vertical tubes. These authors obtained measurements of the film thickness variation with time, over a wide range of Re_f, and used them to improve the fundamental understanding of thin film flow by observations of interfacial waves and by statistical analysis. An indication of the laminar-turbulent transition was observed by them for $Re_f \approx 1500$.

Concerning liquid films falling around Taylor bubbles, the literature is scarce and we can highlight the works of Nogueira *et al*. [11] and Llewellin *et al.* [12]. Results obtained by Nogueira *et al.* [11] suggested the occurrence of the laminar-turbulent transition for $Re_f > 320$. On the other hand, after demonstrating that for thin films the bubble Reynolds number (Re_b) can be considered equal to Re_f, Llewellin et al. [12] obtained indications that this transition would occur for $Re_f > 1000$. The bubble Reynolds number Re_b can be defined as:

$$Reb = \frac{\rho L * Ub * D}{\mu L}, \qquad (2)$$

where, U_b is the rise bubble velocity and D is the inner tube diameter.

Ultrasonic techniques are non-intrusive, which aroused the interest of its application in studies of multiphase flows, since their transducers do not physically interact with the flow. These techniques are low cost and can be used in high pressure and temperature flows. Additionally, ultrasonic techniques can be applied in transparent or opaque fluids inside tubes or containers of different materials.

Chang *et al.* [13] have used a pulse-echo ultrasonic technique to perform a study of liquid height measurement and flow pattern characterization of horizontal two-phase gas-liquid flows. Lu *et al.* [14] have applied a pulse-echo technique to measure the thickness of liquid films generated from the condensation process on the cooled bottom surface of a pipe. These authors asserted that, for the ultrasonic system used by them, the pulse-echo technique could be applied to measure the thickness of liquid films, only if this film was smooth or showed small amplitudes in the interfacial waves. For the measurement of these thicknesses in the presence of more significant interfacial waves, an effort would be required to obtain the signals in digital mode and with a higher data acquisition rate. Since then, ultrasonic systems have evolved and recently we have performed an experimental study of the falling film around a Taylor bubble by using a pulse-echo ultrasonic technique. We measured the bubble velocity and the bubble shape, trying to define if the liquid film really reaches the equilibrium (De Azevedo *et al.* [15]). The results indicated that the development of the film was reached for almost all cases studied, but the equilibrium thickness δ_{eq} was not constant.

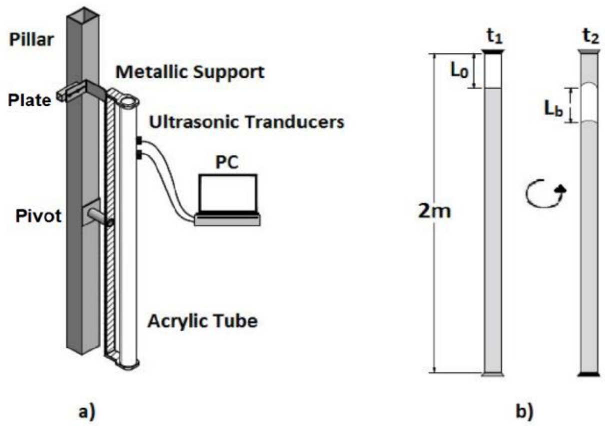
In the present work, a pulse-echo ultrasonic technique was applied to measure the bubble rise velocity U_b and the equilibrium thickness δ_{eq} of liquid films falling around Taylor bubbles rising in stagnant liquid vertical columns sealed at the ends and with different inner diameters D. These parameters together with the calculated standard deviation σ of the equilibrium film thickness provided information about the development of waves on the gas-liquid interfaces. The film development lengths Z* were also determined. Water, glycerin, and water-glycerin solutions were used as working fluids, corresponding to film Reynolds number Re_f ranging from 2 to 7650. The experimental results obtained were used to relate the formation of waves at the gas-liquid interface with the laminar-turbulent transition of the flow of liquid falling around Taylor bubbles.

2. EXPERIMENTAL EQUIPAMENT AND PROCEDURES

The experiments were carried out in vertical columns partially filled with stagnant water-glycerin mixtures. The columns consisted of acrylic cylindrical tubes with 2.0 m in length and inner diameters of 0.019, 0.024 and 0.034 m, sealed at the ends. The tube was partially filled with the liquid, leaving an air pocket of length L_0 and a Taylor bubble with length L_b was formed by its inversion (t_1 to t_2), as illustrated in Fig.2b.

The acrylic tubes were fixed and aligned on a metallic support that can be rotated around a pivot. The tubes were held in the desired position by a small plate that limited the support rotation and allowed their inversion and positioning on the vertical, as presented in Fig.2a. The vertical position was determined by using a digital angle sensor, with an uncertainty of $\pm$ 0.1°.

Figure2: *Schematic of a stagnant vertical column: a) Apparatus; b) Bubble formation.*



Different combinations of inner tube diameters (D = 0.019, 0.024 and 0.034 m) and working liquids (pure distilled water, pure glycerin, and glycerin-water mixtures) were studied. For each of these combinations (D, liquid), at least ten single bubbles were generated. Thus, the results presented correspond to averaged values of, at least, ten measurements of the desired parameters for each of these sets of data.

The fluid properties were calculated by correlations given by Cheng [16] for density (ρ_L) and viscosity (μ_L) of the glycerin-water mixtures in the range of 0 - 100% and temperatures in the range of 0 - 100°C. The experimental conditions are summarized in Table 1.

The high-speed ultrasonic system used in the measurements consisted of a generator-multiplexer board, transducers and a personal computer with a LabVIEW software developed to control up to four transducers in pulse-echo or transmission modes. Two piezoelectric-type ultrasonic transducers of 10 MHz and 6.35 mm diameter (Olympus Model V112) were mounted at 0.75 and 0.80 m from the top of the tube, respectively, at one side of it. The transducers were clamped on the outer surface of the tubes by means of supports, especially designed in order to assure the transducers alignment.

Table1: Experimental conditions.

Liquid	D(m)	T (°C)	ρ_L (kg/m^3)	μ_L (Pa.s)	$Re_f = Re_b$
100%W	0.019	29.0	995.8	0.0008	3510
80%W+20%G	0.019	30.0	1058.3	0.0015	1999
50%W+50%G	0.019	30.0	1141.6	0.0057	570
20%W+80%G	0.019	30.0	1214.1	0.0499	64
100%G	0.019	30.0	1257.4	0.5979	3
100%W	0.024	24.0	997.1	0.0009	4533
80%W+20%G	0.024	26.0	1059.9	0.0017	2526
50%W+50%G	0.024	25.0	1144.2	0.0068	676
20%W+80%G	0.024	26.0	1216.5	0.0629	74
100%G	0.024	24.0	1261.3	0.9875	2
100%W	0.034	26.0	996.6	0.0009	7650
80%W+20%G	0.034	29.0	1058.7	0.0016	4504
50%W+50%G	0.034	28.0	1142.7	0.0061	1305
20%W+80%G	0.034	28.0	1215.3	0.0559	142
100%G	0.034	28.5	1258.7	0.7035	8

The pulse-echo ultrasonic technique is based on the high difference between the acoustic impedance of the gas and liquid phases, which allow that almost 99% of the incident wave be reflected by a gas-liquid interface. Thus, the location of a gas-liquid interface can be determined by measuring the transit time between the emission of a wave and its return after being reflected (Chang and Morala [17]). Details about the pulse-echo ultrasonic technique and the measurements of the desired parameters can be found in De Azevedo et al. [15] and De Azevedo et al. [18].

The equilibrium thickness of the liquid film δ_{eq} is defined by an average value with a small relative discrepancy, which can represent the presence of waves at the gas-liquid interface. This relative discrepancy can be defined by the coefficient of variation (Cv), also known as relative standard deviation:

$$\mathrm{Cv}=\frac{\sigma}{\delta \mathrm{avg}}, \tag{3}$$

where, σ is the standard deviation defined by:

$$\sigma=\sqrt{\frac{1}{N-1}\sum_{\mathrm{i}=1}^{N}(\delta \mathrm{i}-\delta \mathrm{avg})^2} \tag{4}$$

and δ_{avg} is the average value of the film thickness at the developed region (See Fig.1):

$$\delta \mathrm{avg}=\frac{1}{N}\sum_{\mathrm{i}=1}^{N}\delta\, i. \tag{5}$$

In Eqs. (4) and (5), δ_i is the film thickness at the point i of the developed region of the liquid film.

Thus, the thickness on the developed region of the liquid film around a Taylor bubble (δ_{eq}) can be defined as:

$$\delta \mathrm{eq}=\delta \mathrm{avg} \pm \mathrm{Cv}, \tag{6}$$

where Cv and δ_{avg} were determined as described by Eqs.(3) and (5) respectively.

3. RESULTS AND DISCUSSION

The results obtained by ultrasonic measurements of the bubble velocities have already been presented in previous works and showed excellent agreement with simultaneously measurements made by using high speed video camera and with a universal correlation proposed by Viana *et al.* [19] to predict the rise velocity of a Taylor bubble in vertical tubes (De Azevedo *et al.* [15], De Azevedo [20]). The bubble velocities were used in the present work basically to determine the bubble Reynolds number Re_b, according to Eq.(2). In the present work we adopted an approximation proposed by Llewellin *et al.* [12], who showed that for thin films ($\delta << R$), $Re_f = Re_b$. The values of Re_f calculated and used for each experimental condition were presented in Table1.

The dimensionless average thicknesses δ_{avg}/R of the liquid films in the region where they were fully developed are shown in Fig. 3. It can be seen that for small values of Re_f,, a sharp decrease on δ_{avg}/R occurred when Re_f increased. On the other hand, as Re_f continued to increase, the decrease on δ_{avg}/R became much smoother.

Figure 3: *Dimensionless average thicknesses at the developed region of the film vs Re_f.*

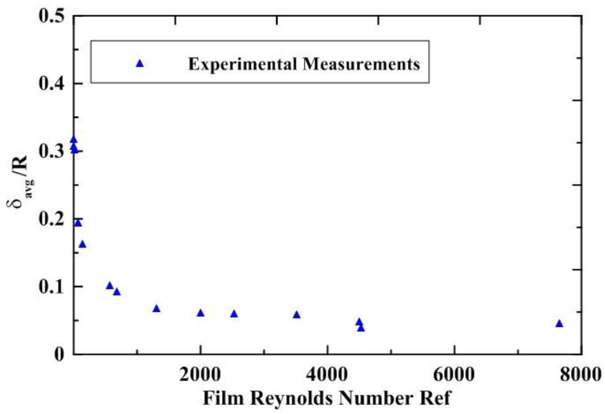
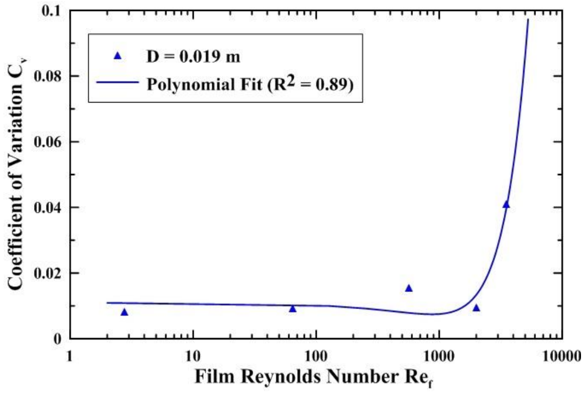
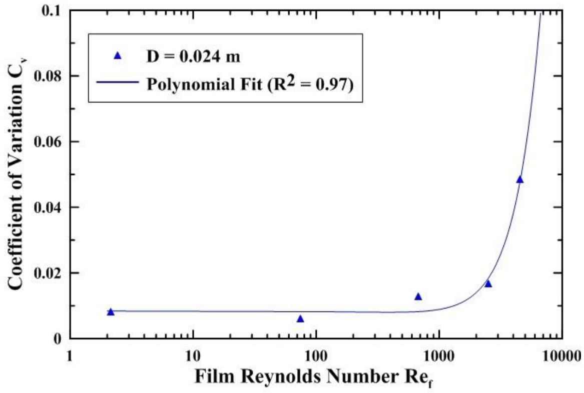


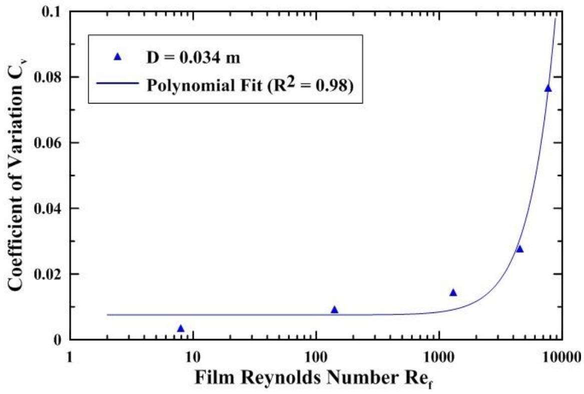
Figure 4: *Coefficients of variation vs Re_f for different liquids inside a tube with: a) D = 0.019 m; b) D = 0.024 m and c) D = 0.034 m.*



a)



b)



c)

The coefficients of variation Cv of the thicknesses at the developed region of the films were calculated for the different working liquids and plotted in Fig. 4 against Re_f for each inner tube diameter D used in the experiments. Fig. 4 shows that for each D studied Cv almost did not change for Re_f up to around 1000 but increased sharply when Re_f increased beyond 1000. The increase on Cv with increasing Re_f could be related to a more intense formation of waves at the gas-liquid interfaces of Taylor bubbles, which suggests a change in the flow regime of the liquid falling around it. In order to better observe the behavior of Cv with Re_f, all calculated values of Cv were plotted in Fig. 5 against Re_f independently of the inner tube diameters D used in the experiments.

Figure 5: *Coefficients of variation vs Re_f for all conditions studied in the present paper.*

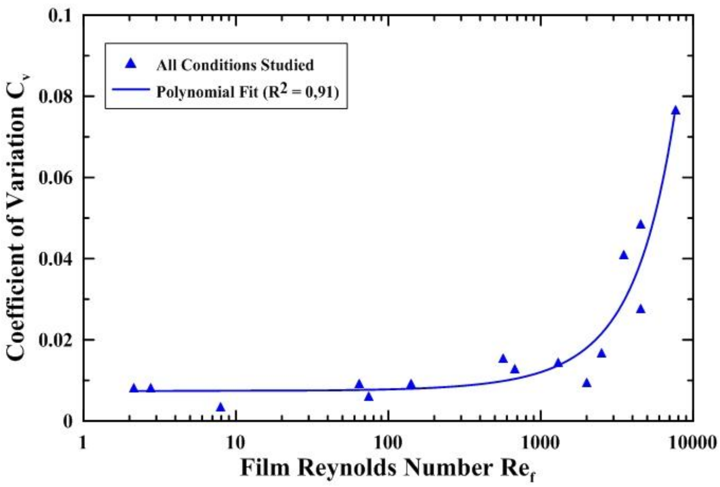


Figure 5 confirms the behavior of Cv with the Re_f increase observed in Figs.4(a)-(c). Up to Re_f around 1000, the coefficient of variation Cv did not change significantly, but a sharp increase on Cv can be observed for Re_f higher than this value. As discussed above, this behavior suggests a change in the flow regime of the liquid falling around the bubbles for values of Re_f around 1000 and this change would be the laminar to turbulent transition. This flow regime transition occurring for values around 1000 can be considered in agreement with the literature previously cited for liquid films falling in surfaces (Dukler and Bergelin [6], Fullford [7], Salazar and Marschall [8], Aragaki *et al*. [9], Karapantsios *et al.* [10]), since all these authors have proposed ranges for the laminar to turbulent transition for values of Re_f close to 1000. Our results would also agree well with Llewellin *et al.* [12] whose experimental results have indicated that the laminar to turbulent transition for films falling around Taylor bubbles would occur for $Re_f > 1000$.

Another evidence that the laminar to turbulent transition would occur for $Re_f \approx 1000$ can be seen in Fig.6 from the comparison between the experimental measurements of the film development length Z* and an expression used to predict this parameter proposed by Sena Esteves and Guedes de Carvalho [21] with the assumption of laminar flow.

Figure 6: *Dimensionless film development lengths Z*/D vs Re_f for all conditions studied in the present paper.*

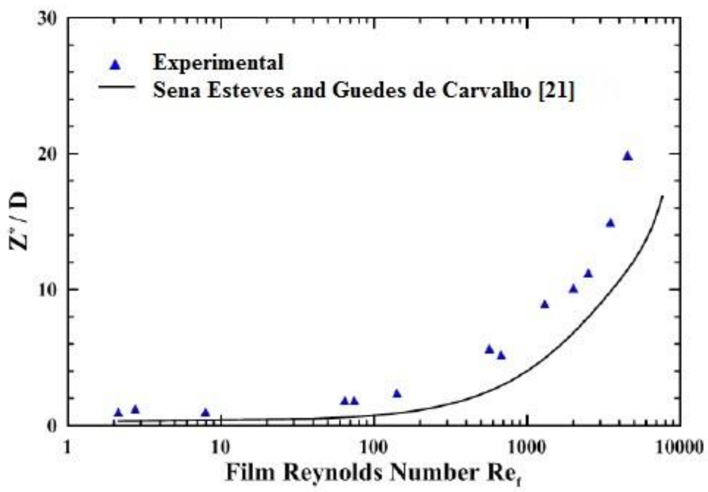


Figure 6 shows that the experimental results agreed relatively well with the theoretical expression for smaller values of Re_f, with differences between the measured values and the predicted ones around 1D. A greater difference between them can only be observed from values of Re_f close to 1000. The disagreement observed from Re_f around 1000 could be attributed to a transition of the film flow regime from laminar to turbulent, since the theoretical expression plotted in the figure was derived with the assumption of laminar flow.

The results obtained in the present work are quite interesting and promising, especially considering the lack of studies available in the literature that address the flow regime transition in liquid films falling around Taylor bubbles. As discussed earlier, only two works (Llewellin *et al.* [12] and Nogueira *et al.* [11]) have attempted to evaluate the transition from laminar to turbulent in the flow of these films and their results were somewhat discordant. However, our results and analysis were in agreement with those presented by Llewellin *et al.* [12], with the laminar to turbulent transition occurring for $Re_f \approx 1000$, which would also be in accordance with the transition range observed by several authors for liquids flowing on surfaces.

Thus, our results suggest that the applied pulse-echo ultrasonic technique proved to be feasible to identify the formation of waves at the gas-liquid interfaces of Taylor bubbles moving in liquid columns and can be used as an interesting tool for a more complete study on the flow regime transition of the liquid film falling around this kind of bubble.

4. CONCLUSION

Based on experimental measurements of the bubble rise velocity U_b and the equilibrium thickness δ_{eq} of liquid films falling around Taylor bubbles by using a pulse-echo ultrasonic technique and on the determination of the coefficient of variation Cv of the film thickness at its developed region, the following conclusions can be presented:

- The dimensionless average thicknesses δ_{avg}/R of the liquid films in the fully developed region decreased sharply when Re_f increased for small values of Re_f. On the other hand, as Re_f continued to increase, δ_{avg}/R decreased more smoothly.

- The increase on Cv when the film Reynolds number (Re_f) increased could be related to an intense formation of waves at the gas-liquid interfaces of Taylor bubbles moving in liquid. Up to values of Re_f around 1000, Cv did not change significantly, but a sharp increase on it can be observed for Re_f higher than this value, which suggests a change in the flow regime on the liquid film falling around the bubbles. This change would be the laminar to turbulent transition and would occur for values of Re_f around 1000.

- A comparison between the experimental measurements of the film development length Z* and an expression used to predict this parameter proposed by Sena Esteves and Guedes de Carvalho [21], with the assumption of laminar flow, also reinforces the occurrence of the laminar to turbulent transition for $Re_f \approx 1000$, since a great difference between the experimental measurements and the theoretical predictions can be observed from Re_f close to this value.

- The results obtained suggest that the pulse-echo ultrasonic technique is feasible to identify the formation of waves at the gas-liquid interfaces of Taylor bubbles moving in liquid columns.

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