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Neutron angular flux reconstruction in slab geometry using multigroup discrete ordinates transport models

Balbina F. T. C. S[a], Alves Filho H.[b], Barros R.C.[b]

[a] *Programa de Pós-Graduação em Engenharia Biomédica*

Universidade Anhembi Morumbi

Centro de Inovação, Tecnologia e Educação - CITÉ

12244-550 São José dos Campos, SP, Brazil

ftcoimbra@gmail.com

[b] *Departamento de Modelagem Computacional, Instituto Politécnico*

Universidade do Estado do Rio de Janeiro

Rua Bonfim, 25, Vila Amélia

28625-570 Nova Friburgo, RJ

halves@iprj.uerj.br

dickbarros.apple@gmail.com

ABSTRACT

In this article, we present an application of the coarse-mesh Deterministic Spectral Method (SDM) to generate multigroup angular fluxes in one-dimensional spatial domains using the neutron transport stationary equation, in the formulation of discrete ordinates (S_N), considering isotropic scattering source. After obtaining the analytical solution of the S_N equations, we replace the integral term of the scattering source in the original neutron transport equation. Thus, we obtain analytically two expressions for angular fluxes in the multigroup formulation, considering the neutron propagation in the positive ($\mu > 0$) and negative ($\mu < 0$) directions, presenting a meaningful reduction in the computational time of simulations of typical neutron shielding problems.

Keywords: **neutron transport theory, discrete Ordinate method, angular reconstruction methodology**

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1. INTRODUCTION

The neutron transport analysis has the physical model based on the migration of these particles inside a material media and on the probability of their interaction with the nuclei of the atoms of the media. The properties of the materials where the neutrons migrate are characterized phenomenologically in terms of the cross sections determined theoretically or experimentally [1,2]. The physical phenomenon of transport of neutral particles in a material medium is of interest in various scientific applications, e.g., nuclear reactors, shielding calculations, radiation protection, nuclear medicine etc. In all, there is a need for an accurate description of the phenomenon of particles transport into material media.

The computational modeling is presented as a tool capable of collaborating to solve the problems that fall into systems of integral equations, partial differential equations or ordinary differential equations of high number of unknowns, making use of numerical methods and construction of models from of simplifying hypotheses to obtain response. The consideration of an approximation of the transport equation dependent on the energy variable is necessary for more realistic calculations of the neutron transport. The multigroup approach, the most common method for discretization of the energy variable, consists in dividing the energy interval into contiguous energy groups [1-3].

The fixed-source problems, also known as shielding calculations (deep penetration problems), mainly involve the transport of particles through coarse "shields" that can cause significant absorption or scattering of neutrons, resulting in changes in the energy and direction of these particles at energy intervals and/or localized spatial regions can be extremely small and consequently a large number of experiments (histories) and computational times are required (compared to deterministic methods) to achieve statistically reliable results if the Monte probability method is used [4,5]. Thus, in this paper we use the deterministic method of discrete ordinates (S_N), proposed by Wick [6] and Chandrasekhar [7], a classic method that continues to be used to solve transport equation in several applications. It has its basis in the discretization of the independent angular variable being the integral of the source term approximated by a numerical quadrature, thus transforming the transport equation into a system of differential equations that can be solved by numerical or analytical approaches [3].

A deterministic approach to neutron shielding problems (source-fixed) has reached great interest in recent years. We can cite as an example the MOC [8,9], cf., Method of Characteristics, the traditional polynomial methods [10-11] and spectral nodal methods, e.g., the SGF method [12-13], cf. spectral Green´s function, the RM method [14], cf., Response Matrix and SDM [15-17], cf., Spectral Deterministic Method, that significantly reduce the execution times of the computational codes used in these models, presenting good accuracy in numerical results.

The spectral nodal methods, a methodology introduced less than 30 years ago [12-13] are algebraic and computationally more laborious than traditional deterministic fine-mesh numerical methods, e.g., the DD, cf., Diamond Difference [3], but present more precision in the solutions numerical values for relatively fine spatial nodes; for this reason, these numerical methods and their possible algorithms for iterative and direct solution schemes have been the object of studies in recent years.

The objective of the work is based on the linearized Boltzmann equation, to consider the multigroup S_N equations in a one-dimensional domain with isotropic scattering and to create a mathematic tool capable of analytically solving the original multigroup neutron transport equations to obtain values for the angular fluxes in any direction, position x and in g energy group. This aim is achieved through an angular reconstruction of the neutron transport equations in the multigroup formulation considering the approximation of angular moments of the scattering source, which is expanded in Legendre polynomials in the angular variable, in the multigroup transport equations, by the angular moments (scalar flux), obtained from the numerical solution of the equations S_N. Thus, the proposal is to solve analytically the neutron transport multigroup equations to obtain values for the angular fluxes in any direction, position x and in any energy group used in the model.

The methodology presented in this work is an extension of that developed in the research [18-20] and can be described in simplified form as follows.

The spectral nodal method SDM [15-17] generates the angular fluxes at the interfaces of a one-dimensional domain, using Gauss-Legendre quadrature set, in discrete ordinates (S_N) multigroup formulation, in the neutron transport theory. Once the coarse-mesh solution was obtained with the SDM method, we first determined the arbitrary constants of the analytical general solution of the S_N

equations, treating the term of isotropic scattering source accurately (without approximations) within each spatial node. We then use the angular quadrature formula to estimate the neutron scalar flux. We solve the one-dimensional multigroup transport equation analytically, with the term of the scattering source thus approximated. As the SDM method generates solutions completely free from spatial truncation error, this analytic reconstruction within in node is quite accurate.

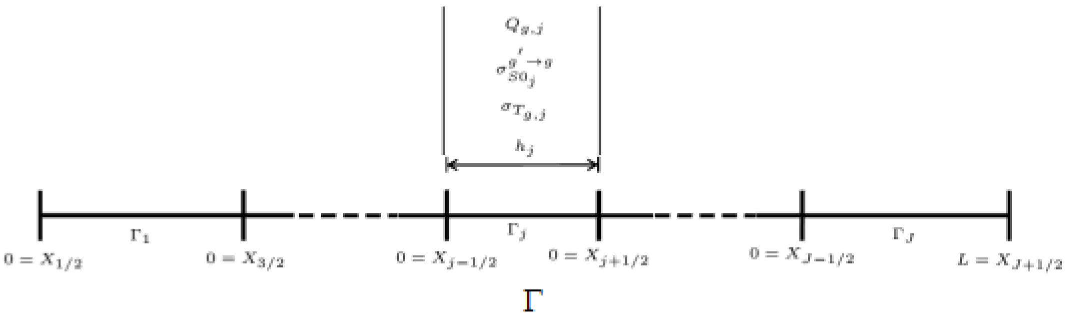
Aim to obtain the numerical results available in this work, a computational application was developed (portuguese language) in the MatLab, which streamlined the preparation and execution of the model case presented.

This paper is organizated in form. In section 2 we describe the mathematical preliminaries about SDM method and presented the angular reconstruction expressions to neutron angular flux in multigroup equations for positive and negative directions. In section 3, numerical results to one model-problem is presented and Section 4 we explained the conclusion and presented the ideas to future papers.

2. MATHEMATIC PRELIMINARIES

Let us consider the stationary multigroup transport equations with isotropic scattering in discrete ordinates formulation described in arbitrary node Γ_j in spatial grid Γ one-dimentional spatial domain D with width L (Figure 1).

Figure 1: *Spatial Grid* Γ.



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$$\mu_m \frac{d}{dx}\psi_{m,g}(x) + \sigma_{Tgj}\psi_{m,g}(x) = \sum_{g'=1}^{G}\frac{\sigma_{S0j}^{g'\to g}}{2}\sum_{n=1}^{N}\omega_n\psi_{n,g'}(x) + Q_{gj}, g = 1:G, m = 1:N. \tag{1}$$

In Eq.(1) $x \in \Gamma$, G represents the number of energy groups and N is the order of the Gauss-Legendre quadrature set [3] used for the solution of the S_N problem. The variable *m* represents discrete direction of neutron propagation. The parameter σ_{Tgj} describes the total macroscopic cross section of the g-th group; which includes all possible interactions and $\sigma_{S0j}^{g'\to g}$ is the zero'th components of the macroscopic differential scattering cross section from group g' to group g, respectively and Q_{gj} is the isotropic external source-fixed in the energy group g. The dependent variable $\psi_{m,g}(x)$ represents the angular flux of particles traveling in the discrete ordinates direction μ_m for each group g and ω_m is the angular weight. The Eq. (1) has the prescribed boundary conditions in form.

$$\psi_{m,g}(x) = \begin{cases} \psi_{m,g}(x_{j-1/2}) = f_{m,g}, \text{if } \mu_m > 0, \\ \psi_{m,g}(x_{j+1/2}) = k_{m,g}, \text{if } \mu_m < 0, \; m = 1:N, g = 1:G \end{cases} \tag{2}$$

In Eq. (2) the values of $f_{m,g}$ and $k_{m,g}$ are known (prescribed).

The general solution within in node of the Eq. (1) is expressed as

$$\psi_{m,g}(x) = \psi_{m,g}^{h}(x) + \psi_{m,g}^{p}, \quad m = 1:N, g = 1:G. \tag{3}$$

The superscript *p* denotes the particular solution with fixed-source and *h* indicates the homogeneous component of the local general solution, which satisfies the system of Eq.(1). The particular solution ψ_{gj}^{p}, with isotropic fixed-source Q_{gj}, appears in system form [17].

$$\sigma_{Tgj}\psi_{m,g}^{p} - \sum_{g'=1}^{G}\frac{\sigma_{S0j}^{g'\to g}}{2}\sum_{n=1}^{N}\omega_n\psi_{m,g'}^{p} = Q_{gj} \tag{4}$$

Now, to determine the homogeneous component $\psi^{h}_{m,g}(x)$, we consider the expression [17]

$$\psi^{h}_{m,g}(x) = a_{m,g}(\vartheta)\exp[-(x - x_{j-1/2})/\vartheta_{\ell}],\ m = 1:N, g = 1:G \ , \tag{5}$$

where $x_{j-1/2}$ represent left node-edge boundary $\Gamma_{\mathbf{j}}$ as shown on Figure 1. Substituting the Eq. (5) in homogeneous part of the Eq. (1), we obtain the eigenvalue problem

$$\sigma_{Tgj}a_{m,g}(\vartheta) - \sum_{g'=1}^{G}\frac{\sigma^{g'\to g}_{S0j}}{2}\sum_{n=1}^{N}\omega_n a_{n,g'}(\vartheta) = \frac{1}{\vartheta}a_{m,g}(\vartheta), m = 1:N, g = 1:G. \tag{6}$$

The Eq (6) can be written in the eigenvalue problem in operators form

$$A\mathbf{a} = \frac{1}{\vartheta}\mathbf{a} \ . \tag{7}$$

Here, the A is a real matrix of the GN x GN order which generates the N eigenvalues ϑ_{ℓ} with ther respective linearly independent eigenvectors $a_{m,g}(\vartheta_{\ell})$. So, the general analytical solution within in node of Eq. (1), appears in form

$$\psi_{m,g}(x) = \sum_{\ell=1}^{NG}\alpha_{\ell}a_{m,g}(\vartheta_{\ell})\exp[-(x - x_{j-1/2})/\vartheta_{\ell}] + \psi^{p}_{m,g}, m = 1:N, g = 1:G. \tag{8}$$

2.1 Espectral Deterministic Method (SDM)

In a very brief way, the SDM method consists of determining the angular outgoing fluxes from the initial estimates of the angular incoming fluxes at the nodes, using Eq. (8). Once the boundary conditions are known, the domain is traversed, starting with the left side (x = 0), node to node, to the right side of the domain (x = L). For each iteration, the following operations are performed:

a) we solve the eigenvalues ϑ_ℓ problem described by Eq. (6), where eigenfunctions $a_{m,g}(\vartheta_\ell)$ are obtained with their respective eigenvalues;

b) the constants α_ℓ are calculated using Eq. (8), considering the angular fluxes incoming in node Γ_j. After calculating the α_ℓ constants we proceed to obtain the angular fluxes also using Eq. (8).

When all nodes of the space domain are crossed over a transport iteration is completed. The angular fluxes at the boundaries of the nodes obtained in the iteration will then be used as initial estimates for the next iteration. The process will be done, until the pre-established convergence criterion is satisfied.

The stop criterion establishes that the maximum norm of the difference of the vector scalar flux, considering two successive iterations, must be smaller than a pre-established value according to expression

$$\max_{j=1:J}\left|\frac{\Psi^{k}_{g,j-1/2}-\Psi^{k-1}_{g,j-1/2}}{\Psi^{k}_{g,j-1/2}}\right|<\varepsilon,\ g=1:G, \tag{9}$$

where $(\boldsymbol{\Psi}^{k}_{g,j-1/2}-\boldsymbol{\Psi}^{k-1}_{g,j-1/2})$ represents the difference between scalar flux vector obtained in the previous iteration $k-1$ and the currently executed iteration k. The component for the neutron scalar flux appears in the form

$$\phi_g(x)=\frac{1}{2}\sum_{n=1}^{N}\omega_n\psi_{n,g}(x), g=1:G. \tag{10}$$

A complete description of the SDM method can be found in references [17-19].

2.2. Angular reconstruction of neutron angular flux in multigroup formulation

With the numerical solutions obtained through the SDM method, as we have seen in the previous section, we have already determined the constants α_ℓ of the local general solution we can determine the analytical solution at each arbitrary node Γ_j, in the discrete directions of the chosen S_N model. From this strategy, we can develop a methodology for obtaining the analytical solutions of the original neutron transport equation. The essence of this process is to approximate the angular moments of the scattering source, which is expanded in Legendre polynomials in the angular variable, in the transport equation, by the angular moments (angular flux) obtained from the numerical solution of the S_N equations seen in Eq (8). With these approximations made in terms of scattering source, the proposal is to solve analytically the multigroup neutron transport equations to obtain an equation that provides values for the angular fluxes in any angular direction μ, position x and any group g of energy used in the model. To perform this procedure, we consider the original multigoup neutron transport equation [3] in arbitrary node Γ_j (Figure 1).

$$\mu\frac{\partial}{\partial x}\psi_g(x,\mu)+\sigma_{Tgj}\psi_g(x,\mu)=\sum_{g'=1}^{G}\frac{\sigma_{S0j}^{g'\to g}}{2}\int_{-1}^{+1}\psi_{g'}(x,\mu')d\mu'+Q_{gj},$$
$$x=0:L, g=1:G, \mu\in[-1,0)\cup(0,+1]. \tag{11}$$

Eq. (11), in spatial node Γ_j, has the prescribed boundary conditions in form

$$\psi_g(x,\mu)=\begin{cases}\psi_g(x_{j-1/2},\mu)=b_g(\mu), \text{ if } \mu>0,\\ \psi_g(x_{j+1/2},\mu)=c_g(\mu), \text{ if } \mu<0,\ g=1:G.\end{cases} \tag{12}$$

In Eq. (12) the values of b_g and c_g are known (prescribed).

The integral term of Eq. (11), considering Eq. (8), can be written approximated as

$$\int_{-1}^{+1}\psi_{g'}(x,\mu)d\mu'\cong\sum_{n=1}^{N}\omega_n\psi_{n,g'}(x)=\sum_{n=1}^{N}\omega_n\left[\sum_{\ell=1}^{NG}\alpha_\ell a_{n,g'}(\vartheta_\ell)\exp[-(x-x_{j-1/2})/\vartheta_\ell)+\psi_{n,g'}^{p}\right]. \tag{13}$$

Now, substituting the Eq. (13) in integral term of the Eq. (11), we obtain

$$\mu \frac{\partial}{\partial x} \psi_g(x,\mu) + \sigma_{Tgj} \psi_g(x,\mu) =$$
$$\sum_{g'=1}^{G} \frac{\sigma_{S0j}^{g' \to g}}{2} \left(\sum_{\ell=1}^{NG} \sum_{n=1}^{N} \omega_n \alpha_\ell \, a_{n,g'}(\vartheta_\ell) \exp[-(x - x_{j-1;2}) / \vartheta_\ell] + \sum_{n=1}^{N} \omega_n \psi_{n,g'}^{p} \right) + Q_{gj}, \; g = 1:G. \tag{14}$$

Using the integral factor $\exp(\sigma_{Tgj} x / \mu)$ and dividing the whole equation by μ after some algebra one arrives at the form

$$\frac{\partial}{\partial x} \{\psi_g(x,\mu) \exp(\sigma_{Tgj} x / \mu)]\} =$$
$$\sum_{g'=1}^{G} \frac{\sigma_{S0j}^{g' \to g}}{2\mu} \left[\sum_{\ell=1}^{NG} \sum_{n=1}^{N} \omega_n \alpha_\ell \, a_{n,g'}(\vartheta_\ell) \exp[\frac{(-\mu + \sigma_{Tgj} \vartheta_\ell) x}{\mu \vartheta_\ell} - \frac{x_{j-1/2}}{\vartheta_\ell}] + \sum_{n=1}^{N} \omega_n \psi_{n,g'}^{p} \exp(\sigma_{Tgj} x / \mu) \right] + \tag{15}$$
$$Q_{gj} \exp(\sigma_{Tgj} x / \mu), \; g = 1:G.$$

Now, from Eq. (15), we will determine analytical solution as follows. For the positive directions of the neutron propagation $(\mu > 0)$, the Eq. (15) is integrated in the interval $\int_{x_{j-1/2}}^{x} (.) dx$ and after some algebra takes the form

$$\psi_g(x,\mu) = \psi_g(x_{j-1/2},\mu) \exp(-\sigma_{Tgj}(x - x_{j-1/2}) / \mu) +$$
$$\sum_{g'=1}^{G} \frac{\sigma_{S0j}^{g' \to g}}{2} [\sum_{\ell=1}^{NG} \sum_{n=1}^{N} \frac{\vartheta_\ell \omega_n \alpha_\ell a_{n,g'}(\vartheta_\ell)}{-\mu + \sigma_{Tgj} \vartheta_\ell} \left[\exp \frac{-(x - x_{j-1/2})}{\vartheta_\ell} - \exp(\frac{-\sigma_{Tgj}(x - x_{j-1/2})}{\mu}) \right] +$$
$$\sum_{n=1}^{N} \frac{\omega_n \psi_{n,g'}^{p}}{\sigma_{Tgj}} \left[1 - \exp(\frac{-\sigma_{Tgj}(x - x_{j-1/2})}{\mu}) \right]] + \frac{Q_{gj}}{\sigma_{Tgj}} \left\{ 1 - \exp(\frac{-\sigma_{Tgj}(x - x_{j-1/2})}{\mu}) \right\}, \tag{16}$$
$$g = 1:G, x = 0:L, \; \mu \in [-1,0).$$

For the negative propagation directions $(\mu < 0)$ the Eq. (15) is integrated in the interval $\int_{x}^{x_{j+1/2}} (.)\,dx$ and, after some algebra we obtain

$$\psi_g(x,\mu) = \psi_g(x_{j+1/2},\mu)\exp(-\sigma_{Tgj}(x - x_{j+1/2})/\mu) +$$

$$\sum_{g'=1}^{G} \frac{\sigma_{S0j}^{g' \to g}}{2} [\sum_{\ell=1}^{NG}\sum_{n=1}^{N} \frac{\vartheta_\ell \omega_n \alpha_\ell a_{n,g'}(\vartheta_\ell)}{-\mu + \sigma_{Tgj}\vartheta_\ell} \left[\exp\frac{-(x - x_{j-1/2})}{\vartheta_\ell} - \exp(-\frac{h_j}{\vartheta_\ell} - \frac{-\sigma_{Tgj}(x - x_{j+1/2})}{\mu}) \right] + \tag{17}$$

$$\sum_{n=1}^{N} \frac{\omega_n \psi_{n,g'}^{p}}{\sigma_{Tgj}} \left[1 - \exp(\frac{-\sigma_{Tgj}(x - x_{j+1/2})}{\mu}) \right]] + \frac{Q_{gj}}{\sigma_{Tgj}} \left\{ 1 - \exp\frac{-\sigma_{Tgj}(x - x_{j+1/2})}{\mu} \right\},$$

$$g = 1:G, x = 0:L, \mu \in (0,+1]$$

In conclusion, Equations (16) and (17) represent the angular reconstruction of the angular flow in the positive and negative directions.

3. NUMERICAL RESULTS AND DISCUSSION

In order to demonstrate the validity of the methodology developed in this paper were used one characteristic model-problem. The main objectives, with the use of model-problem, were to compare the results obtained by the SDM method against results obtained to angular reconstruction see in Eq. (16) and Eq. (17). We obtain the results for the angular flux reconstruction using a low quadrature order, reconstruct directions for the propagation of higher order quadrature, taking into account the Percentage Relative Deviation (PRD) less than 1%, calculated with expression

$$PRD = \left| \frac{V_{SDM} - V_{REC}}{V_{SDM}} \right| * 100, \tag{18}$$

where V_{SDM} represents the value obtained by SDM method and V_{REC} the value obtained for angular reconstruction, using the Eq. (16), for $(\mu > 0)$ or Eq.(17), for $(\mu < 0)$.

3.1. Model-problem

The model-problem [21] is a homogenous domain with length of 10 cm, with G = 19 groups and isotropic fixed-source Q = 0 for all groups. It is a 10 cm (one region) iron plate with boundary conditions prescribe to isotropic angular fluxes incoming equal a unit (x= 0) in the first (g = 1) of the nineteen (G = 19) energy groups in range of 50 keV to 1 MeV and vacuum boundary condition in x = 10 cm. The material parameters is presented in reference [21].

The Table 1 presented the results obtained by the SDM for the scalar neutron flux for some energy groups chosen arbitrarily, compared to those shown in the reference [22], which used the coarse-mesh spectral method SGF. For the solution of the problem we use Gauss-Legendre quadrature order N = 4, with $\varepsilon = 10^{-6}$ and one node per region. We emphasize here that the use of the SGF method was to validate the SDM method, implemented in the computational application.

Table 1: Scalar fluxes (cm^{-2} s^{-1}).

Group	**SGF** [a]		**SDM**	
(g)	**0 cm**	**10 cm**	**0 cm**	**10 cm**
1	5.079466e-01 [b]	7.400293e-04	5.079466e-01	7.400293e-04
4	1.205968e-02	1.184823e-04	1.205968e-02	1.184823e-04
7	9.329249e-03	1.001940e-04	9.329249e-03	1.001940e-04
10	1.364114e-02	1.487979e-04	1.364114e-02	1.487979e-04
12	2.375113e-02	2.418206e-04	2.375113e-02	2.418206e-04
15	5.814921e-03	8.959714e-05	5.814921e-03	8.959714e-05
17	4.101255e-04	6.735422e-06	4.101255e-04	6.735422e-06
19	4.048850e-06	6.705707e-08	4.048850e-06	6.705707e-08

[a] Numerical results generated from SIMFAT [21].
[b] Read as 5.079466 x 10^{-1}.

The numerical results obtained for the scalar fluxes using the SDM are the same as those found in the reference [22] using the SGF. The SDM method, like the SGF, generates solutions free from spatial truncation errors in the interfaces of the domain, independently of the number of nodes used by region.

We present in Table 2 the numerical results to the angular reconstruction of the neutron angular fluxes in x = 5 cm (half the domain), using the order of the quadrature $N = 6$ (S_6), to calculate the α_ℓ coefficients of Eq. (16), positive $(\mu > 0)$ and Eq. (17), negative $(\mu < 0)$ directions of high order quadrature set S_{8}, S_{16}, S_{32}, S_{64} and S_{128}. For the reconstruction of the directions in high quadrature directions, the results were satisfactory, being below 1% deviation relative in relation of the results the SDM method, which we consider acceptable for the experiment. The angular directions and energy group values were chosen at random mode. In that same experiment, the angular reconstruction technique was calculated using the quadratute order N = 4, but in some cases relative deviation RD was above the 1% stipulated as the maximum limit of the experiment presented.

Table 2: Angular flux reconstruction (cm^{-2} s^{-1}) to x = 5 cm and N = 6.

Relation	Position (cm)	Angular Direction (μ)	Group (g)	Angular Flux (V_{SDM})	Angular Flux Eq. (16) (V_{REC})	Angular Flux Eq. (17) (V_{REC})	PRD (%)
$S_6 \to S_8$	5	μ1 = + 0.1834346	5	0.0020786	0.0020814	-----	0.13
		μ3 = + 0.7966665	11	0.0066193	0.0066171	-----	0.08
		μ6 = - 0.5255324	15	0.0026083	-----	0.0026109	0.10
		μ8 = - 0.9602899	19	0.0000051	-----	0.0000051	0.00
$S_6 \to S_{16}$	5	μ4 = + 0.6178762	17	0.0003700	0,0003707	-----	0.19
		μ6 = + 0.8656312	14	0.0050979	0,0051053	-----	0.15
		μ10 = - 0.28160356	3	0.0010381	-----	0.0010453	0.69
		μ13 = - 0.7554044	9	0.0014360	-----	0.0014429	0.48
$S_6 \to S_{32}$	5	μ4 = + 0.3318686	4	0.0022828	0.0022872	-----	0.19
		μ8 = + 0.6630443	10	0.0047291	0.0047370	-----	0.17
		μ25 = - 0.7321821	14	0.0028237	-----	0.0028337	0.35
		μ30= - 0.9647623	18	0.0000284	-----	0.0000285	0.35
$S_6 \to S_{64}$	5	μ8 = + 0.3572202	16	0.0011752	0.0011781	-----	0.25
		μ27 = + 0.9610088	3	0.0056533	0.0056651	-----	0.21
		μ47 = - 0.6489655	2	0.0009284	-----	0.0009354	0.75 [a]
		μ61 = - 0.98333623	8	0.0010444	-----	0.0010500	0.54
$S_6 \to S_{128}$	5	μ15 = + 0.3471177	1	0.0023726	0.0023743	-----	0.07
		μ44 = + 0.87405278	7	0.0036537	0.0036655	-----	0.32
		μ81 = - 0.39254023	12	0.0044858	-----	0.0045057	0.44
		μ108 = - 0.87405278	15	0.0024015	-----	0.0024097	0.34

[a] Maximum Percentage Relative Deviation.

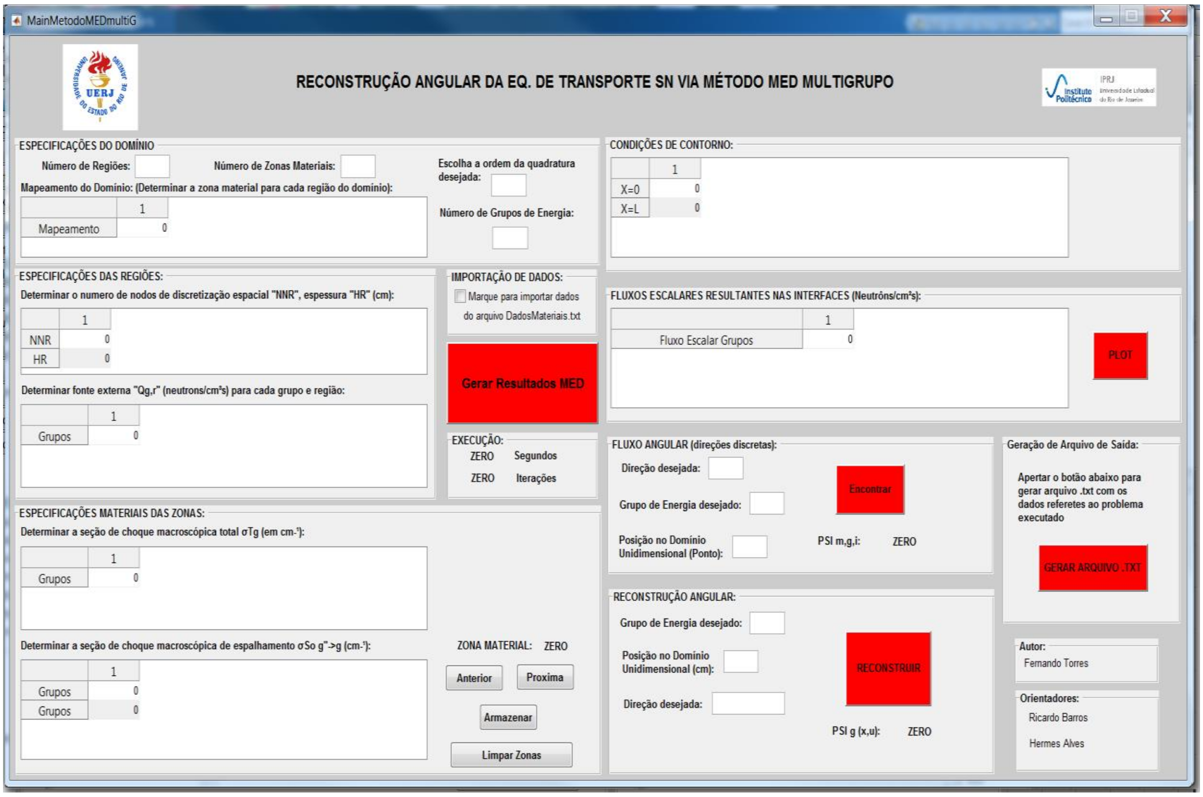
In Table 3 we show in a simplified way the reduction of the execution time of the simulations seen in Table 2. As can be seen, there was a considerable reduction in the execution time in the angular reconstructions with considerable precision of the presented results. This fact generates a future perspective for the simulation of more complex problems, considering multidimensional cases, arbitrary degrees of anisotropy and multigroup in energy.

Table 3: Execution time (s).

Relation	**Execution Time (V_{SDM}) (s)**	**Execution Time Eq. (16) or Eq.(17) (V_{REC}) (s)**
$S_6 \rightarrow S_8$	2.04	0.35 (V_{REC})
$S_6 \rightarrow S_{16}$	8.09	
$S_6 \rightarrow S_{32}$	32.92	
$S_6 \rightarrow S_{64}$	131.99	
$S_6 \rightarrow S_{128}$	2270.98	

In Figure 3, we show the main screen of the computational application (Simulator) developed in the MatLab language. From this screen it is possible to access another screen´s necessary to obtain the numerical results presented in Table 1, Table 2 and Table 3. More details on the operation of this application can be found in the reference [23].

Figure 2: *Main Screen of Computational Simulator.*



Source: Authors

4. CONCLUSIONS

In this paper, we present a numerical technique capable of analytically modeling the original neutron transport multigroup equations to obtain the values for the angular fluxes in angular direction μ (positive and negative), position x and *g* energy group, considering models in one-dimensional spatial domains, with the isotropic scattering macroscopic cross section. Using the SDM coarse-mesh numerical solution, we first determine the arbitrary constants α_{ℓ} of the analytical general solution of the S_N equations within each spatial discretization node; then we obtain an expression for the multigroup scalar flux that we substitute into the isotropic scattering source term in the original neutron transport equations. We then solve analytically the slab-geometry multigroup original transport equation in order to generate analytically two expressions to neutron angular fluxes in multigroup formulation, considering the neutron propagation in positive ($\mu > 0$) and negative ($\mu < 0$) directions.

The angular reconstruction strategy, using Eq. (16) and (17), for model-problem, allowed us to conclude that the results were satisfactory, reaching a relative deviation below 1%, a value considered acceptable by us in the simulation of this type of problem. The reduction of the CPU computational cost through angular reconstruction was significant when considering the problems with low quadrature order in the calculation of the alpha ((α_{ℓ}) parameters to estimate the angular fluxes in directions present in quadratures with higher orders that demand a higher CPU computational cost to be obtained.

Here, we also used a Gauss-Legendre quadrature generator of arbitrary order, which made it easier to obtain the data for the simulations presented in the numerical results. Therefore, after analyzing the numerical results, we conclude the validity of the methodology presented here.

We propose for future work the extension of this methodology to problems multigroup with arbitrary degree of anisotropy and rectangular Cartesian X. Y-geometry.

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