BJRS

BRAZILIAN JOURNAL
OF
RADIATION SCIENCES
09-01A (2021) 01-12

SBPR
Sociedade Brasileira de Proteção Radiológica

Source-detector effective solid angle calculation using MCNP6 code

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ABSTRACT

Nuclear techniques based on the attenuation of gamma radiation are used in the industry to calculate flow rate, determine fluid density, predict inorganic scale in oil pipelines, evaluate industrial mixers, among other applications. In order to use these nuclear techniques is necessary to perform studies of important parameters of radioactive source and radiation detectors, which are part of the measurement geometry, such as detection efficiency and solid angle. The aim of this study is to calculate the solid angle and the effective solid angle subtended by a NaI(Tl) detector. The effective solid angle considers attenuation in the medium (between source and detector) and other effects of radiation interaction with matter. Mathematical models were developed using the MCNP6 code in order to evaluate the proposed measurement geometry. The source was placed in different positions to the detector to evaluate frontal and lateral solid angle contributions, which is an important parameter to obtain the intrinsic efficiency response function. The simulated model consists of a NaI(Tl) scintillator detector and two point isotropic sources (^{241}Am and ^{137}Cs). The results for the geometry used in this study showed that the difference between solid angle and effective solid angle reached 20.17% for ^{241}Am and 2.58% for ^{137}Cs, which means that it is highly recommended to consider the effective solid angle in the calculations.

Keywords: **effective solid angle, NaI(Tl) detector, MCNP6 code.**

ISSN: 2319-0612
Accepted: 2020-12-08

1. INTRODUCTION

The solid angle is an important parameter of measurements geometries and has been studied in many configurations of source-detector for many years [1 – 5]. In the nuclear field, in applications involving gamma radiation detectors, the intrinsic efficiency of a detector and the solid angle that is subtended by a detector are required to estimate the emission rate of the gamma radiation source for example. In this case, the detection efficiency is lower than 100%, which makes necessary the determination of this parameter to correctly record the number of photons incident on a detector in a given energy from a radiation source. If a photon rate (N) is recorded, knowing the intrinsic efficiency (ε_i) and the solid angle (Ω), then the emission rate of the isotropic source (A) can be estimated, disregarding scattering and attenuation, as $A=4\pi N/\Omega\varepsilon_i$ [6]. The efficiency can be classified in two types: intrinsic and absolute efficiencies.

The intrinsic efficiency (ε_i) of a detector, which depends on intrinsic characteristics of the detector, can be defined as the ratio of the number of photons registered by the number of photons incident on the detector. In NaI(Tl) scintillation detectors, the atomic number of the sensitive element and the physical state of the detector material are some of these intrinsic characteristics that strongly influence the efficiency. Meanwhile, the absolute efficiency (ε_a) can be defined as the ratio between the number of photons registered and the number of photons emitted by the radiation source. The two efficiencies are related by the solid angle (Ω) of the detector as seen by the position of the radiation source. In a simplified way, Ω can be defined as a cone of view from a point to a certain object. Nevertheless, the effective solid angle (Ω_E) subtended by a detector can be defined as the amount of particles emitted by the source entering the detector aperture and it requires the study of attenuation in the medium and others effects of the radiation interaction with matter.

Although the analytical calculation of the solid angle is not complicated when using simpler geometries with point sources placed at a collinear axis of the cylindrical detector. The difficulty on analytical calculation of the solid angle in complex geometries is expected [7 – 9]. However, the Monte Carlo method can be used to deal with a variety of geometries and overcome this calculation difficulty [1 – 4, 10, 11]. In this way, this study proposes the use of MCNP6 code [12], based on the Monte Carlo method, to evaluate the solid angle and the effective solid angle subtended by a

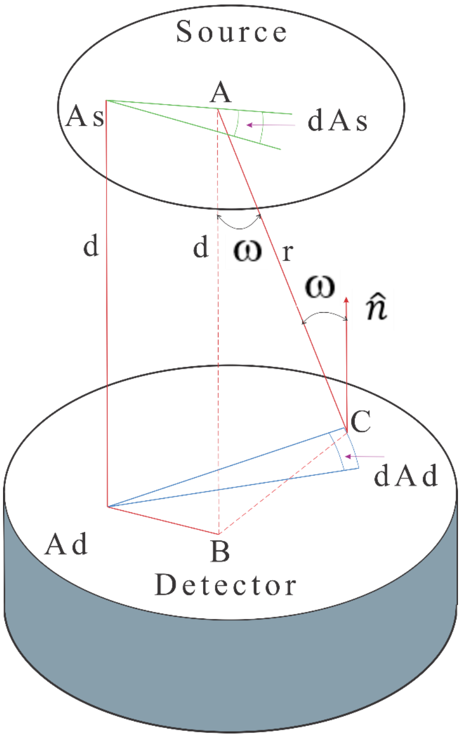
NaI(Tl) detector. In the first moment, the methodology is evaluated comparing the MCNP6 code with both analytical equations and theoretical results found in the literature in a geometry where the point source is placed on the collinear axis of a detector. Then, more complex geometries, where the source is placed outside the collinear axis of the detector using point source, are investigated using the code and validated with data from the literature. Finally, the solid angle calculation methodology using the MCNP6 code is tested in a simplified model of a NaI(Tl) detector using a ^{241}Am (59.5 keV) and ^{137}Cs (662 keV) point isotropic sources and varying the aluminum thickness of the detector casing. In addition, in order to evaluate the uncertainty in the calculation of the solid angle in a possible positioning error of the radiation source, small variations in the position of the source were simulated.

2. SOLID ANGLE

Generally, the solid angle (Ω) subtended by a detector is defined as the number of particles per second emitted within the space defined by the source and the detector aperture contours divided by the number of particles per second emitted by the source [6]. The mathematical definition of the solid angle is obtained as follows Figure 1. Consider a plane source with “A_s” area, emitting “S” particles/(m^2.s) isotropically located at distance “d” from a detector with “A_d” aperture. For this case, the Equation 1 gives the Ω [6]:

$$\Omega = \frac{\int_{A_s}\int_{A_d}\left(S dA_s/4\pi r^2\right) dA_d (\hat{\mathbf{n}}\cdot\mathbf{r}/r)}{S\cdot A_s} \tag{1}$$

Figure 1: *Solid angle for a plane source and a detector with circular aperture.*



Source: Adapted from Tsoulfanidis, 1983 [6].

It is possible to obtain a simplified mathematical expression for the solid angle considering a point isotropic source in a certain collinear distance “d” from the detector with a circular aperture and radius “R”. Since $\hat{\boldsymbol{n}} \cdot \boldsymbol{r}/r = cos\omega$ and considering $cos\omega = d/r$, Equation 1 can be solved and then Equation 2 gives the **Ω** of a point isotropic source [13]:

$$\Omega = 2\pi\left(1 - \frac{d}{\sqrt{d^2+R^2}}\right) \tag{2}$$

As the Equation 1 can be solved analytically only in a few cases, one way to overcome this problem is using the Monte Carlo method. In the MCNP6 code [12], the command tally F1 gives particle current without regard for the charge of the particles and when multiplied by the source emission (4-π) it calculates the solid angle subtended by a detector in a given position.

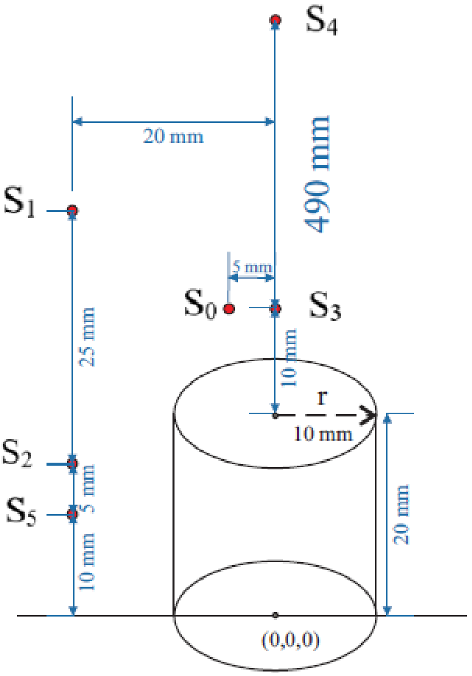
3. METHODOLOGY

The simulations of this study were carried out using MCNP6 code [12], which is a mathematical code based on the Monte Carlo method. MCNP6 code considers the main effects of radiation interaction with matter, such as photoelectric effect, Rayleigh and Compton scattering. The number of starting particles (NPS) was set to 1E8 in order to stablish a relative error below 1% in the simulations, which is an acceptable value by the user's manual [12].

3.1. Solid Angle of a Cylindrical Detector

The first step of this study was to compare the MCNP6 code simulations with both analytical equations and theoretical results found in the literature. These cases were simulated using a ^{137}Cs (662 keV) point isotropic source (4-π) and an upright circular cylinder. The detector has 10 mm of radius and 20 mm of length. The source is placed in six different positions (S_0, S_1, S_2, S_3, S_4 and S_5) as follows Figure 2.

Figure 2: *Different positions of the point isotropic source.*



3.2. Solid Angle of a NaI(Tl) Detector

In the second step of this study, solid angle cases were performed using a 2 × 2 cm NaI(Tl) detector. The detector model is simplified and it considers the NaI(Tl) crystal as a homogeneous cylinder with a cylindrical aluminum casing. To observe behavior of the study solid angle (Ω) and effective solid angle (Ω_E) in different energies, two point isotropic sources were studied: ^{137}Cs (662 keV) and ^{241}Am (59.5 keV). The sources were placed in the same positions described in Figure 2 (S_0 to S_5) to study Ω and Ω_E. In order to study the influence of materials used in the simulation in the effective solid angle calculations, frontal and lateral thickness of the aluminum casing of the detector were varied (1 mm and 3 mm). In addition, small variations in the positioning of the source were studied, in positions S_1, S_3 and S_5, to observe the influence of these variations in the solid angle and in the effective solid angle, i.e., these variations aimed to observe how small displacements in the source influence the solid angle values of the detector. The three positions were varied ±1 mm assuming a situation in which has been made a positioning error in the radiation source. It was only considered to move the radiation source close to or away from the detector.

4. RESULTS AND DISCUSSION

This section presents the results of solid angle and effective solid angle obtained in this study. Relative error (RE) is a measure of the uncertainty of measurement compared to the reference value. This is good way to measure the difference between two values. RE (%) is calculated using Equation 3:

$$RE\ (\%) = \left(\frac{|X_{ref} - X|}{X_{ref}}\right) \times 100 \tag{3}$$

Where:

X_{ref} is the reference value;

X is the obtained value.

4.1. Solid Angle of a Hypothetical Detector

The solid angle (Ω) values of source positions in the collinear axis of the detector (S_3 and S_4) were compared to the analytical reference value (Equation 2), as follows Table 1.

Table 1:*Values of solid angle calculated by MCNP6 code and by Equation 2.*

Position	**Ω MCNP6**	**Ω Equation 2**	**RE(%)**
S_3	1.8395	1.8403	0.0436
*S_4	0.0012	0.0012	0

*S_4 difference in Ω value was beyond fourth decimal place.

The values of the solid angles calculated by means of the MCNP6 and compared to the literature are in Table 2. Column Ω_1 represents the values of F1 tally multiplied by 4π (MCNP6 code results); column Ω_2 are the values obtained by Doron (2007) [2]; column Ω_3 are the values obtained by Wielopolski (1977) [1] and column Ω_4 are the values obtained by Masket et al. (1956) [5]. It is important to note that in positions S_0, S_3 and S_4 the detector was set to a disk shape.

Table 2:*Values of solid angle calculated by MCNP6 code and results found in the literature.*

Position	**Ω_1 (This paper)**	**Ω_2 [2]**	**Ω_3 [1]**	**Ω_4 [5]**	**RE (%) Ω_4 - Ω_1**
S_0	1.6366	-	1.6572	1.6371	0.0305
S_1	0.3792	0.3807	0.3775	0.3791	0.0264
S_2	1.2624	1.2623	1.2602	1.2624	0
S_3	1.8395	-	-	1.8403	0.0435
*S_4	0.0012	-	-	0.0012	0
S_5	1.3901	-	-	1.3899	0.0144

*S_4 and S_2 difference in Ω value was beyond fourth decimal place.

RE(%) was calculated in order to compare the results of the MCNP6 code (Ω_1) and the table of solid angles ORNL-2170 (Ω_4) [5], which is a complete reference of solid angle calculation. These results indicate that the proposed methodology is able to calculate the solid angle in agreement with the values found in the literature.

4.2. Solid Angle of a NaI(Tl) Detector

Since the methodology to calculate the solid angle (Ω) for a point isotropic source and a hypothetical detector with a circular aperture is well established, it is possible to calculate the Ω subtended by a NaI(Tl) detector. In order to observe attenuation in the medium (between source and detector) and other effects of radiation interaction with matter, the effective solid angle (Ω_E) was calculated, thus it was possible to compare Ω_E with Ω using point isotropic sources of ^{241} Am and ^{137}Cs. The frontal and lateral thickness of the aluminum casing of the detector was varied (1 mm and 3 mm) in positions S_0 to S_5. The results are presented in Table 3 and the comparison of Ω_1 (value from Table 2) and Ω_E is given by relative error (RE).

Table 3: *Effective solid angle subtended by a NaI(Tl) detector using ^{241}Am and ^{137}Cs.*

Positions	**1 mm of Al thickness**				**3 mm of Al thickness**			
	^{241}Am		**^{137}Cs**		**^{241}Am**		**^{137}Cs**	
	Ω_E	**RE(%)**	**Ω_E**	**RE(%)**	**Ω_E**	**RE(%)**	**Ω_E**	**RE(%)**
S_0	1.5464	5.51	1.6242	0.76	1.3820	15.56	1.6003	2.22
S_1	0.3488	8.02	0.3750	1.11	0.3027	20.17	0.3694	2.58
S_2	1.1829	6.30	1.2523	0.80	1.0581	16.18	1.2389	1.86
S_3	1.7407	5.37	1.8261	0.73	1.5605	15.17	1.8003	2.13
***S_4**	0.0012	0.00	0.0012	0.00	0.0011	8.33	0.0012	0.00
S_5	1.3041	6.19	1.3793	0.78	1.1682	15.96	1.3643	1.86

* S_4 is 500 mm away from the detector and significant changes in Ω and Ω_E were beyond fourth decimal place.

It is noteworthy that the distance source-detector was not changed in this simulation with Al thickness, thus the solid angle is the same presented in Table 2 (Ω_1). The results show that the greater the thickness of the detector casing, higher the radiation attenuation, which led to an increase in the relative error when comparing the solid angle with the effective solid angle. In addition, it is possible to highlight that for low energies (^{241}Am – 59.5 keV) the difference between the solid angle and the effective solid angle is greater than for high energies (^{137}Cs – 662 keV). Since the definition of the effective solid angle considers the radiation interaction with materials, such as: air (between source and detector), and the materials of the detector itself.

In Table 3, it is possible to notice that position S_1 presents the highest difference between solid angle and effective solid angle (worst case: 20.17% for ^{241}Am and 3 mm of Al thickness). This could be explained because, as the source is displaced from the collinear axis of the detector, the path taken by the radiation in the attenuator material is greater. The same effect is noticed observing positions S_0 and S_3. Considering low energy (^{241}Am) and 3 mm of Al thickness, when moving the radiation source from position S_3 to position S_0, effective solid angle is reduced from 1.5605 (S_3) to 1.3820 (S_0), which is a reduction of 11%, approximately. For position S_1, this attenuation is higher, probably due frontal and lateral contributions in the solid angle and effective solid angle calculations.

Next step of this solid angle study is to observe how small displacements of the radiation source influence the solid angle values. In this way, S_1, S_3 and S_5 were chosen to make these small variations in the positioning of the radiation source, varying ±1 mm simulating an error in the radiation source positioning. S_1 and S_5 were varied horizontally (to the left the source moves away and to the right the source approaches the detector) and S_3 was varied vertically (upwards the source moves away and downwards the source approaches the detector). The results of this study are presented in Table 4, which RE_1 represents the relative error of the solid angle and RE_2 represents the relative error of the effective solid angle. RE_1 and RE_2 were calculated in relation to the original position.

Table 4: *Results of small variation in the radiation source position.*

Position	Ω	**RE_1 (%)**	**1 mm of Al thickness**				**3 mm of Al thickness**			
			^{241}Am		**^{137}Cs**		**^{241}Am**		**^{137}Cs**	
			Ω_E	**RE_2 (%)**	Ω_E	**RE_2 (%)**	Ω_E	**RE_2 (%)**	Ω_E	**RE_2 (%)**
S_1 (left)	0.3677	3.04	0.3377	3.17	0.3635	3.07	0.2915	3.70	0.3575	3.21
S_1 (right)	0.3909	3.09	0.3602	3.27	0.3867	3.13	0.3146	3.94	0.3816	3.30
S_3 (up)	1.6334	11.20	1.5475	11.10	1.6218	11.19	1.3896	10.95	1.5992	11.17
S_3 (down)	2.0793	13.03	1.9648	12.87	2.0636	13.01	1.7576	12.63	2.0340	12.98
S_5 (left)	1.2523	9.91	1.1756	9.85	1.2427	9.90	1.0538	9.79	1.2290	9.92

S_5 (right)	1.5494	11.46	1.4524	11.37	1.5371	11.44	1.3002	11.30	1.5209	11.47

When small variations are made in radiation source positioning, in position S_3 down, the solid angle value reached 13.03% of difference in relation to the original position S_3. In addition, by varying the aluminum thickness of the detector casing by 1 mm or 3 mm, it was possible to observe only small variations in the calculation of the effective solid angle.

5. CONCLUSION

In this study, it was possible to establish that the MCNP6 code is a great tool to perform solid angle calculation, including complex geometries outside of the collinear axis of the detector. In measurements of radiation detection, when speaking about solid angles it is necessary to consider the effective solid angle especially when it is required to calculate the intrinsic efficiency of a radiation detector. This study showed that the difference between solid angle and effective solid angle reached 20.17% for ^{241}Am (59.5 keV) and 2.58% for ^{137}Cs (662 keV) (both with 3 mm of Al thickness and both at position S_1), which means that it is highly necessary to consider the effective solid angle mainly at low energies. This result could be explained because, as the source is displaced from the collinear axis of the detector, the path taken by the radiation in the attenuator material is greater. In the study on small variations in the positioning of the radiation source, solid angle value reached 13.03% of difference in position S_3 down in relation to the original position S_3. In addition, it was possible to note that these variations in the radiation source positioning were not significant by varying the aluminum thickness of the detector casing by 1 mm or 3 mm in the effective solid angle calculations.

ACKNOWLEDGMENT

The authors gratefully acknowledge scholarship from Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq) and Coordenação de Aperfeiçoamento de Pessoal de Nível Superior - Brasil (CAPES) – Finance Code 001. The authors also thank Instituto de Engenharia Nuclear (IEN).

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