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A brief history of Accident Chernobyl: Simulation of the influence of Neutron Absorbing Poisons and temperature feedback effects by Point Kinetics Equations

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ABSTRACT

In this paper, the solution of the Neutron Point Kinetics model is presented, adding the effects of temperature and absorbers poisons within a historical and technical context to simulate the preliminary characteristics of the Chernobyl accident. The Point Kinetics model was able to extract physical information consistent with what was expected to predict the reactor situation until the accident. It was also possible to verify, given the results, that the Rosenbrock method was able to overcome the degree of stiffness of the ODE system, besides solving a non-linear problem. Thus, this study has contributed to highlighting the importance of temperature effects and especially absorbers poisons in the final power behavior, extremely relevant for decision making in the operation and safety of a nuclear power plant.

Keywords: **Neutron Point Kinetics Equations, Temperature feedback, Absorbers poisons, Chernobyl accident simulation, Rosenbrock method.**

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1. INTRODUCTION

The Chernobyl nuclear accident occurred on April 26th, 1986, at the Chernobyl Nuclear Power Plant (originally named Vladimir Lenin) in Ukraine (then part of the former Soviet Union). It is considered the worst nuclear accident in the history of nuclear power, producing a cloud of radioactivity that reached the Soviet Union, Eastern Europe, Scandinavia, and the United Kingdom.

The Chernobyl Vladimir Lenin atomic power plant is located about 183 km from the city of Kiev, the capital of Ukraine, about 20 km from the city of Chernobyl, and 4 km from the city of Pripyat. The plant had four reactors of the RBMK-1000 (Reaktor Bolshoy Moshchnosti Kanalnyy) type each generating about 1000 megawatts of electricity. In addition, two more units were under construction on the day of the accident in 1986, making a total of 6 reactors at the plant. However, in unit 4, there were still safety tests to be completed on the cooling capacity in the power outage, requested by the State Committee for the Use of Atomic Energy [1]. The test consisted of simulating a possible blackout in the system to analyze whether the turbine, driven by the inertia of the residual steam, would be sufficient to keep the water pumps running until the diesel generators were started.

In the early afternoon hours of April 25th, 1986, power is scheduled to be reduced for the safety test of the plant's number 4 reactor. According to the test program, the power should be between 700 MWt to 1000 MWt to start the test [2]. Nevertheless, the test did not take place due to power demand and had to be postponed.

There are two contradictory official theories about the cause of the accident. The first theory exclusively blamed the plant operators [3]. The second theory was published in 1991 and attributed the accident to defects in the design of the RBMK reactor, specifically the control rods [4]. Both theories were strongly supported by different groups, including the reactor designers, Chernobyl plant personnel, and the government. Some independent experts now believe that neither theory was completely right.

While it is known that the reactor had a dangerously positive void coefficient and a more significant reactor defect was the design of the control rods, an overlooked factor by the operators was that the test did not take place due to power demand and had to be postponed. Meanwhile, the reactor operated at half power capacity for an extended period. Due to the delay, Xenon-135, a high neutron absorber poison, accumulated, causing the reactor to almost shut down due to the effects of this negative reactivity. With the delay in response time of the reactor due to contamination, numerous procedures and safety standards were violated to regain the power to perform the test. The neglect of these effects triggered a series of events that led to reactor instability and a power excursion.

In the literature, some works simulate the Chernobyl accident, especially in the final seconds of the power excursion, where one can highlight Fletcher et al. [5], Yoshida et al. [6] and Chan et al. [3]. In the article by Geer et al. [7], a hypothesis about the interpretation of the explosions that occurred at Chernobyl reactor 4 is presented, in which they consider that the first explosion consisted of thermal neutron mediated nuclear explosions in one or rather a few fuel channels, which caused a jet of debris that reached an altitude of some 2500 to 3000 m. The second explosion would then have been the steam explosion most experts believe was the first one. One of the pillars of corroborating evidence is that a group at the V. G. Khlopin Radium Institute in then Leningrad, detected newly produced, or fresh, xenon fission products, 370 km north of Moscow and far away from the major track of Chernobyl debris ejected by the steam explosion and subsequent fires [7].

In this sense, it is important to have prior knowledge of the behavior of the neutron flux when there is poisoning by neutron absorbers elements for the efficient and safe resumption of a nuclear reactor, which must be done gradually until the poisons are consumed. For this, mathematical models are used that are capable of predicting the neutron behavior relevant to the operational control of a plant.

For a realistic simulation, the ideal model should consider 3 spatial dimensions, thermohydraulic effects, the neutron absorbers poisons, delayed neutron precursors, temperature effects, etc. However, the goal of our work is not to evaluate the spatial distribution of the neutron flux in the reactor core, but to observe the behavior with its time evolution taking into account the poisons, delayed neutron precursors and temperature. We also want to show that a simple point kinetics model, considering only the variables mentioned, is already able to predict the occurrence of the Chernobyl accident. For a study considering spatial distribution as well as thermal-hydraulic effects we suggest consulting, for example, Fletcher et al. [5], Yoshida et al. [6], Chan et al. [2], Parisi [8] and Geer et al. [7].

The Neutron Point Kinetics Equations (NPKE) form a set of ordinary differential equations that describes the temporal behavior of the neutron density and the concentration of delayed neutron precursors. There are several papers published in the literature on NPKE such as those by Nahla [9], Aboanber et al. [10] and Mohideen Abdul Razak and Rathinasamy [11]. Some authors solved them by considering temperature feedback effects as the works of Aboanber and Hamada [12], Sathiyasheela [13] and also with the effects of the main neutron absorbing poisons as one can mention the work of Paganim et al. [14].

The Point Kinetics model has a fundamental characteristic called stiffness. This happens due to the large difference in the lifetimes of the prompt and delayed neutrons. This characteristic can restrict the use of some numerical methods for solving the system. In general, methods considered implicit are used to solve equations with this type of problem due to their region of unlimited stability. The Rosenbrock method has been satisfactory for the solution of stiff problems. In the work of Yang and Jevremovic [15], the Rosenbrock method is used to solve the proper Neutron Point Kinetics equations with different types of reactivity insertions, in which the results are compared with reference data in the literature, considered a benchmark in reactor physics. In previous works [16-18], we showed that the Rosenbrock method effectively solves the Neutron Point Kinetics equations considering also temperature effects and neutron absorbing poisons, that is, it is capable of solving nonlinear and stiffness equations efficiently.

In this paper, the results of the behavior of neutron absorbers poisons, temperature, and neutron density are presented to simulate the preliminary characteristics of the Chernobyl accident for each know stage. This is done by solving the Neutron Point Kinetics model coupled to the effects of the main absorbers poisons and temperature, using known accident parameters, through the Rosenbrock method.

The specific objective of this work is to analyze whether the negligence of the effects of neutron absorbers poisons have had a key role in triggering the accident. It is worth mentioning that in this study the thermo-hydraulic effects of the system are not considered. We are concerned with showing the strong influence that this initial activity of suffocation (delay) of the reactor caused by the effects of the poisons led to the complete destabilization of the reactor core leading up to the accident through a simple coupled model, considering the evolution in time of the reactor power, delayed neutron precursors, the main neutron absorbers poisons and temperature. Thus, we will show that a point kinetics model with the mentioned variables is already able to predict the occurrence of the Chernobyl accident.

2. MATERIALS AND METHODS

In reactor kinetics, the neutron flux remains approximately constant in space and energy during a transient, that is, $\phi(r,E,t_1) \simeq \phi(r,E,t_2)$, in certain situations. One can estimate a “form” function $F(r,E)$ to find a function that depends only on time, which is called the “amplitude”' function, $\Gamma(t)$. As a result of this simplification, the Neutron Point Kinetics model is obtained, which describes the temporal behavior of the neutron density and the concentration of delayed neutron precursors, important for operational control and reactor safety.

To simulate the Chernobyl accident, the effects of the main neutron absorbers poisons are coupled to the Point Kinetics model in addition to the insertion of temperature feedback into the system. Two main poisons, Xenon-135 and Samarium-149 are considered because these have relevant neutron absorption cross-sections. As a comparison, we can see from Table 1 that the absorption cross section value (thermal neutrons) of some isotopes, for example plutonium - 239, becomes insignificant close to the cross section value of Xenon -135 and Samarium - 149.

Table 1: Absorption cross sections of some isotopes

Element	Cross Sections (barns)
Tório - 232	6.54
Plutonium - 239	973
Uranio – 238	2.42
Sodium - 23	0.472
Xenon – 135	2.64×10^6
Samarium - 149	6.15×10^4

Source: Adapted from the book: Fundamentals of Nuclear Reactor Physics [19]

The radioactive decay chains of each of these elements are analyzed to construct equations for their concentrations.

In this sense, coupling to the equations of Neutron Point Kinetics the equations of the neutron absorbers poisons concentrations, together with a temperature feedback insertion, one has the complete model defined by Eq. (1). Without loss of generality, considering only one delayed neutron precursor group:

$$\frac{dP(t)}{dt} = \frac{\rho_0 - \alpha[T(t) - T(0)] - \beta}{\Lambda} P(t) + \lambda C(t) - \sigma_{a,Xe} C_{Xe}(t) v P(t) - \sigma_{a,Sm} C_{Sm}(t) v P(t),$$

$$\frac{dC(t)}{dt} = \frac{\beta}{\Lambda} P(t) - \lambda C(t),$$

$$\frac{dC_I(t)}{dt} = \gamma_I \Sigma_f v P(t) - \lambda_I C_I(t),$$

$$\frac{dC_{Xe}(t)}{dt} = \gamma_{Xe} \Sigma_f v P(t) + \lambda_I C_I(t) - \lambda_{Xe} C_{Xe}(t) - \sigma_{a,Xe} C_{Xe}(t) v P(t), \qquad (1)$$

$$\frac{dC_{Pm}(t)}{dt} = \gamma_{Pm} \Sigma_f v P(t) - \lambda_{Pm} C_{Pm}(t),$$

$$\frac{dC_{Sm}(t)}{dt} = \lambda_{Pm} C_{Pm}(t) - \sigma_{a,Sm} C_{Sm}(t) v P(t),$$

$$\frac{dT(t)}{dt} = HP(t),$$

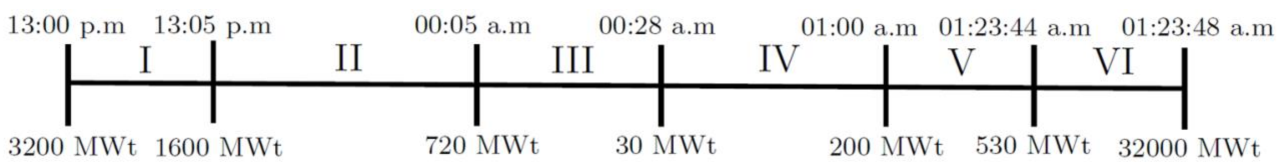
where P here is the neutron density representing a given reactor power level given in $[MWt]$, ρ_0 is the reactivity, β is the total fraction of delayed neutrons, Λ given in $[h]$ is the mean generation time between neutron birth and subsequent collision, λ given in $[h^{-1}]$ is the delayed neutron precursor decay constant, $C(t)$ given in $[cm^{-3}]$ is the concentration of delayed neutron precursors at time t, α is the temperature coefficient, $T(t)$ given in $[K]$ is the temperature at time t, H represents a parameter associated to the influence of heat flux change on the rate of temperature change, v given in $[cm/h]$ is the velocity of the neutron, Σ_f given in $[cm^{-1}]$ is the macroscopic cross-section of fission, $\sigma_{a,Xe}$ given in $[cm^2]$ is the microscopic cross-section of absorption of the element Xenon-135, $C_{Xe}(t)$ given in $[cm^{-3}]$ is the concentration of Xenon-135 at time t, $\sigma_{a,Sm}$ given in $[cm^2]$ is the microscopic cross-section of absorption of Samarium-149, $C_{Sm}(t)$ given in $[cm^{-3}]$ is the concentration of Samarium-149 at time t, γ_I is the fission yield of the nuclide Iodine-135, λ_I given in $[h^{-1}]$ is the radioactive decay constant of Iodine-135, $C_I(t)$ given in $[cm^{-3}]$ is the concentration of Iodine-135 at time t, γ_{Xe} is the fission yield of the nuclide Xenon-135, λ_{Xe} given in $[h^{-1}]$ is the radioactive decay constant of Xenon-135, γ_{Pm} is the fission yield of the nuclide Promethium-149, $C_{Pm}(t)$ given in $[cm^{-3}]$ is the concentration of Promethium-149 at time t and λ_{Pm} given in $[h^{-1}]$ is the radioactive decay constant of Promethium-149. For the system of equations given in Eq. (1), the following initial conditions are considered:

$$
\begin{aligned}
P(0) &= P_0 \,, \\
C(0) &= \frac{\beta}{\lambda \Lambda} P_0 \,, \\
C_I(0) &= \frac{\gamma_I \Sigma_f v}{\lambda_I}, \\
C_{Xe}(0) &= \frac{\gamma_{Xe}\Sigma_f v + \lambda_I C_I(0)}{\lambda_{Xe} + \sigma_{a,Xe} v} \,, \\
C_{Pm}(0) &= \frac{\gamma_{Pm}\Sigma_f v}{\lambda_{Pm}} \,, \\
C_{Sm}(0) &= \frac{\lambda_{Pm} C_{Pm}(0)}{\sigma_{a,Sm} v} \,, \\
T(0) &= T_0 \,.
\end{aligned} \tag{2}
$$

2.1. Estimate for the reactivity

From the known powers of the accident, as illustrated in Fig. 1, it was proposed estimates for the reactivity in each interval. The proposal is to use the Inverse Point Kinetic equations to determine them.

Figure 1: *Chernobyl accident intervals*



It should be noted that the effects of poisons and temperature are disregarded to build the estimates of the reactivities in each interval due to the complexity of the problem. It should also be noted that the variable P here is the neutron density that represents a given power level of the reactor.

Starting from the Inverse Point Kinetics equation [19], one has:

$$
\begin{aligned}
\rho(t) &= \beta + \frac{\Lambda}{P(t)}\frac{dP(t)}{dt} - \frac{\Lambda}{P(t)}\sum_{j=1}^{6} \lambda_k C_k(t) \,, \\
\frac{dC_k(t)}{dt} &= \frac{\beta_k}{\Lambda} P(t) - \lambda_k C_k(t), k = 1,2,\cdots,6 \,.
\end{aligned} \tag{3}
$$

For each interval mentioned in Fig. 1, an associated power function is calculated. The power is assumed to have an exponential shape:

$$P_1 = P_0 e^{wt}, \tag{4}$$

where P_1 and P_0 are, respectively, the power values at the end and beginning of each interval, w is defined as $1/\tau$, where τ is the reactor period given in hours.

The functions found in the intervals I through VI are:

$$\begin{gathered} P_I = 3200e^{-8.6643t}\,, \\ P_{II} = 1600e^{-0.1297378t}\,, \\ P_{III} = 720e^{-8.363721t}\,, \\ P_{IV} = 30e^{3.579452t}\,, \\ P_V = 200e^{2.564635t}\,, \\ P_{VI} = 530e^{3728.56t}\,. \end{gathered} \tag{5}$$

2.2. Rosenbrock's method

The system Eq. (1) has the characteristic of stiffness due to the large difference in the lifetimes of the prompt and delayed neutrons. In this sense, it is more appropriate to use numerical methods that are considered implicit due to their region of unlimited stability [20]. These linearly implicit methods are characterized with excellent linear stability properties [21].

In this sense, Rosenbrock's method was chosen, not only for the above characteristics but also for its ability to solve nonlinear equations by solving a sequence of linear systems instead of using iterative processes.

Thus, the system described in Eq. (1) is solved by the fourth-order, four-stage Rosenbrock method.

At each time step, the solution of Eq. (1) is given by

$$y(t_0 + h) = y(t_0) + b_1 s_1 + b_2 s_2 + b_3 s_3 + b_4 s_4\,, \tag{6}$$

where y represents the dependent variables of Eq. (1), t_0 is the initial time, h is the time step size, $b'_i s$ are the coefficients of the Rosenbrock method, and $s'_i s$ are the vectors corresponding to the stage of the method.

The structure of the fourth-order, four-stage Rosenbrock method used in this paper is an adaptation of the scheme presented in Yang and Jevremovic [15]:

$$
\begin{aligned}
Bs_1 &= f(y_0, t_0) + hc_1 \frac{\partial f(y_m, t_m)}{\partial t} \, , \\
Bs_2 &= f(y_0 + a_{21}k_1, t_0 + h) + \left[hc_2 \frac{\partial f(y_m, t_m)}{\partial t} + \frac{c_{21}k_1}{h} \right] , \\
Bs_3 &= f(y_0 + a_{31}k_1 + a_{32}k_2, t_0 + h) + \left[hc_3 \frac{\partial f(y_m, t_m)}{\partial t} + \frac{c_{31}k_1 + c_{32}k_2}{h} \right] , \\
Bs_4 &= f(y_0 + a_{31}k_1 + a_{32}k_2, t_0 + h) + \left[hc_4 \frac{\partial f(y_m, t_m)}{\partial t} + \frac{c_{41}k_1 + c_{42}k_2 + c_{43}k_3}{h} \right] ,
\end{aligned} \tag{7}
$$

where $B = [(1/\Omega\, h)I - \partial f\,(y_m, t_m)/\partial\, y]$, I is the identity matrix $N \times N$, h is the size of the integration step, Ω is one of the roots of the Laguerre polynomial, $\partial f\,(y_m, t_m)/\partial\, y$ is the Jacobian matrix at $t_{m+1} = t_m + h$ and y_m is the problem evaluated at $t = t_m$ for each m of the mesh.

To calculate the $s'_i s$ vectors, it requires solving four linear systems using the same B matrix for each time step. The coefficients (a, c, b) are real quantities and fixed constants independent of the problem [22]. The determination of the coefficients required by Rosenbrock's method can be obtained from the so-called condition equations reported in Kaps and Rentrop [23].

Since the condition equations are fewer in number than the unknowns, several of them can be chosen as free parameters in order to determine a complete set of constants [22]. The choice of the coefficients of the S-stage Rosenbrock method is determinant as they allow to obtain the order of accuracy of the integration formula and its numerical stability.

In the work of Aboanber and Hamada [22] a study of the stability of the “Rosenbrock generalized Runge-Kutta method” for third- and fourth-order can be found. Also found in Aboanber [24] is a detailed analysis of the convergence and stability of the Rosenbrock method in which the determination of the coefficients required by the method is obtained from the condition equations which, in turn, are derived from an approach based on the Butcher series.

Table 2: Order conditions for Rosenbrock formulas

Order	Order condition
1	$\sum_{i} b_i = 1$
2	$\sum_{i} b_i \beta_i = \frac{1}{2} - \Omega$
3	$\sum_{i} b_i \alpha_i^2 = \frac{1}{3}$
	$\sum_{i,j} b_i \beta_{ij} \beta_j = \frac{1}{6} - \Omega + \Omega^2$
4	$\sum_{i} b_i \alpha_i^3 = \frac{1}{4}$
	$\sum_{i,w} b_i \alpha_i \alpha_{iw} \beta_k = \frac{1}{8} - \frac{1}{3}\Omega$
	$\sum_{i,w} b_i \beta_{iw} \alpha_k^2 = \frac{1}{12} - \frac{1}{3}\Omega$
	$\sum_{i,w,l} b_i \beta_{iw} \beta_{wl} \beta_l = \frac{1}{24} - \frac{1}{2}\Omega + \frac{3}{2}\Omega^2 - \Omega^3$

Source: Adapted from Kaps and Rentrop [23]

Where in Table 1 $i, j, w, l = 1, \ldots, S$. The following abbreviations are considered: $\beta_{ij} = \alpha_{ij} + \Omega_{ij}$, $\alpha_i = \Sigma\alpha_{ij}$, $\beta_i = \Sigma\beta_{ij}$ e α_{ij}=Ω_{ij}=0 for $i \leq j$.

2.3. Method Implementation

Rewriting the system of ordinary differential equations (ODEs) presented in Eq. (1), considering the Inverse Point Kinetics equation (Eq. (3)) and the estimates of the powers (Eq. (5)), in matrix form, one obtains:

$$\frac{dY(t)}{dt} = \boldsymbol{AY} + \boldsymbol{MY} \quad (8)$$

where

$$\boldsymbol{Y} = \begin{pmatrix} P(t) \\ C(t) \\ C_I(t) \\ C_{Xe}(t) \\ C_{Pm}(t) \\ C_{Sm}(t) \\ T(t) \end{pmatrix}.$$

The matrices **A** and **M** carry the linear and nonlinear contributions respectively, and are given by

$$\boldsymbol{A} = \begin{pmatrix} \frac{1}{P}\frac{dP}{dt} + \frac{\alpha}{\Lambda}T_0 & \lambda & 0 & 0 & 0 & 0 & 0 \\ \frac{\beta}{\Lambda} & -\lambda & 0 & 0 & 0 & 0 & 0 \\ \gamma_I \Sigma_f v & 0 & -\lambda_I & 0 & 0 & 0 & 0 \\ \gamma_{Xe}\Sigma_f v & 0 & \lambda_I & -\lambda_{Xe} & 0 & 0 & 0 \\ \gamma_{Pm}\Sigma_f v & 0 & 0 & 0 & -\lambda_{Pm} & 0 & 0 \\ 0 & 0 & 0 & 0 & \lambda_{Pm} & 0 & 0 \\ H & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix},$$

$$\boldsymbol{M} = \begin{pmatrix} 0 & \frac{-\lambda}{\Lambda}P(t) & 0 & -\sigma_{a,Xe}vP(t) & 0 & -\sigma_{a,Sm}vP(t) & \frac{-\alpha}{\Lambda}P(t) \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\sigma_{a,Xe}vP(t) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\sigma_{a,Sm}vP(t) & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

The Jacobian matrix $\boldsymbol{J} = \partial f\ (y_m, t_m)/\partial\ y$ associated with the system of differential equations in Eq. (1), again considering the Inverse Point Kinetics equation (Eq. (3)) and the estimates of the powers (Eq. (5)), is given by

$$\boldsymbol{J} = \begin{pmatrix} \frac{1}{P}\frac{dP}{dt} + \frac{(-\alpha[T(t)-T(0)]-\lambda C(t))}{\Lambda} & \lambda & 0 & -\sigma_{a,Xe}vP(t) & 0 & -\sigma_{a,Sm}vP(t) & \frac{-\alpha}{\Lambda}P(t) \\ \frac{\beta}{\Lambda} & -\lambda & 0 & 0 & 0 & 0 & 0 \\ \gamma_I\Sigma_f v & 0 & -\lambda_I & 0 & 0 & 0 & 0 \\ \gamma_{Xe}\Sigma_f v - \sigma_{a,Xe}vP(t) & 0 & \lambda_I & -\lambda_{Xe} - \sigma_{a,Xe}vP(t) & 0 & 0 & 0 \\ \gamma_{Pm}\Sigma_f v & 0 & 0 & 0 & -\lambda_{Pm} & 0 & 0 \\ -\sigma_{a,Xe}vP(t)0 & 0 & 0 & 0 & \lambda_{Pm} & -\sigma_{a,Sm}vP(t) & 0 \\ H & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

The values of the method parameters used are [15]: $\Omega = 0.5$, $\alpha_{21} = 2$, $\alpha_{31} = 1.92$, $\alpha_{32} =$

0.24, $c_{21} = -8$, $c_{31} = 14.88$, $c_{32} = 2.4$, $c_{41} = -0.896$, $c_{42} = -0.4326$, $c_{43} = -0.4$, $b_1 = 19/9$, $b_2 = 0.5$, $b_3 = 25/108$, $b_4 = 125/108$, $c_1 = 0.5$, $c_2 = -1.5$, $c_3 = 2.42$, $c_4 = 0.116$.

3. RESULTS AND DISCUSSION

In this section, the numerical results for the simulation of the Chernobyl accident are presented through the methodology presented in the previous sections. It should be noted that the events were divided into six intervals (as illustrated in Fig. 1) and, in each of them, the results obtained by the mathematical models are explained by confronting them with the corresponding real situation.

The kinetic parameters used are: $\beta = 0.0056$, $\lambda = 291.4840044797h^{-1}$, $\Lambda = 10^{-8}h$, adapted from Chan et al. [2] for a group of precursors. The nuclear parameters referring to the neutrons poisons are found in the study of Paganim [14]. With respect to temperature, the proportionality constant between temperature and neutron density $H = 2.5 \times 10^{-6}\, K/MWs$ and temperature coefficient of reactivity, $\alpha = 2 \times 10^{-6}K^{-1}$, found in Fletcher et al. [5], are admitted.

The imposed initial conditions consider a reactor initially without the presence of the neutron absorbers poisons, average operating temperature of an RBMK-1000 reactor, $T_0 = 500K$, initial thermal power $P_0 = 3200MWt$ and, finally, initial concentration of the neutron precursors $C_0 = 3200\,\beta/\Lambda\,\lambda cm^{-3}$.

The results of the behavior of the thermal power, delayed neutron precursor concentration, fission product poison concentrations, and temperature for each interval on the timeline are presented in Fig. 2 to Fig. 7. Due to the large variation in the numerical scales of the graphs, the simulations of the intervals are presented independently, assuming the results of the previous interval as initial conditions for the next interval.

In the early afternoon hours of April 25th, 1986, power is scheduled to be reduced to conduct the safety test of the plant's number 4 reactor. According to the test program, the power should be between $700MWt$ to $1000MWt$ to start the test [2]. However, the test did not take place on this day due to power demand and had to be postponed. Meanwhile, the reactor was operating at half power capacity.

Fig. 2 shows the behavior of the events in the first interval. It can be noted that the thermal power, item (a), decays exponentially, but it does not reach exactly the value in the literature, about $1600MWt$

[1], since the thermal-hydraulic effects of the system are not considered here. However, it is observed that the graph of the power obtains a physical behavior consistent with what is expected, decaying over time.

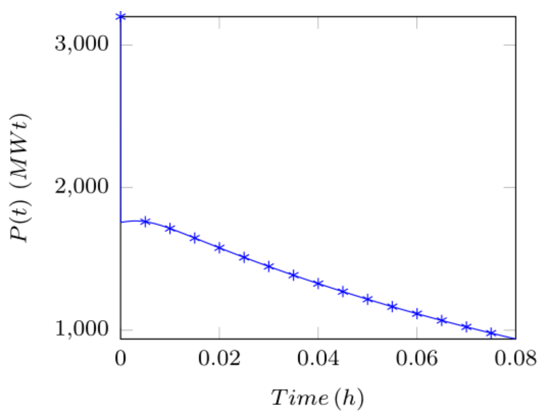
The fission product poisons, on the other hand, have exponential growth. Iodine-135, as well as Promethium-149, according to their variations, arise through nuclear fission and decay through their radioactive decays. The elements Xenon-135 and Samarium-149 are produced through nuclear fission and also through the decay of the other elements (Xenon-135 "is born" through the radioactive decay of Iodine-135 and Samarium-149 through Promethium-149) and decay through their radioactive decays and also through neutron absorption. As the reactor was operating at half power capacity for a long period, Xenon-135, being a strong neutron absorber, has not decayed completely, contaminating the core and contributing to the reduction of reactor power.

Samarium-149, like Xenon-135, also increases its concentration inside the nucleus, and because it is a strong neutron absorber, it again contributes to the decrease in power. The temperature value, in turn, remains practically constant during the first few hours.

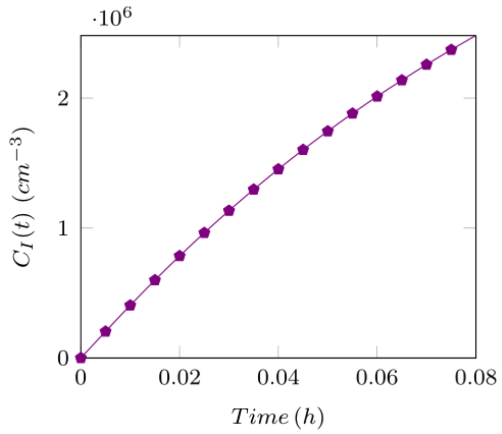
In the second simulation interval, about 11 hours of reactor operation at half the power capacity, the reactor reaches a power of $720MWt$. In Figure 3 item (a), one can see the power reduction still. Due to the continuous decrease in power, it is inferred by items (b) and (d), that the elements Iodine-135 and Promethium-149 begin to suffer a decrease in the value of their concentrations precisely because of the decrease in the number of fissions within the reactor core. Otherwise, we have the element Xenon-135 "being born" still through the radioactive decay of Iodine-135 and Samarium-149 through the decay of the element Promethium-149.

In Fig. 4 items (b) and (d), one notices that the concentrations of the elements Iodine-135 and Promethium-149 continue to decrease. At this time, a peak in the values of the concentrations of the elements Xenon-135 and Samarium-149 begins to be reached, remembering that these elements are strong neutron absorbers.

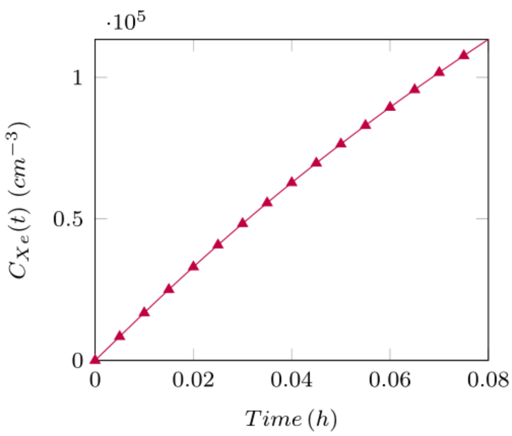
Figure 2: *First interval of simulation (13:00 to 13:05 p.m.)*



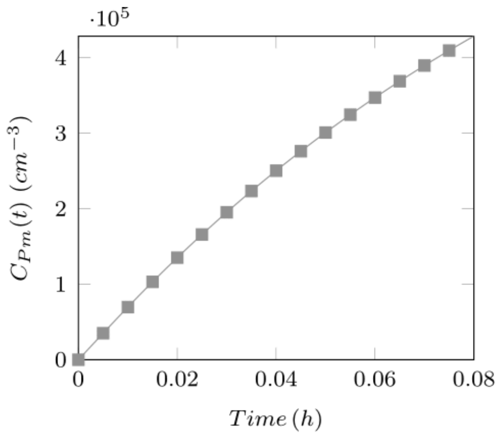
(a) *Time versus* $P(t)$



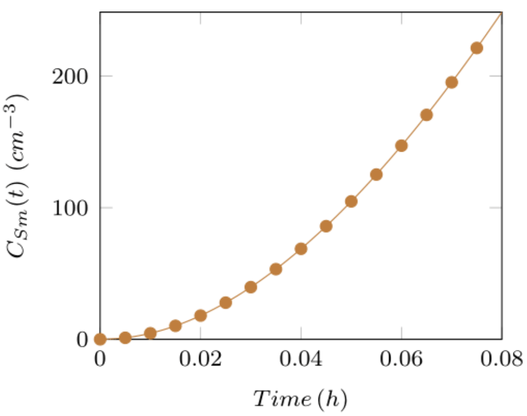
(b) Time versus $C_I(t)$



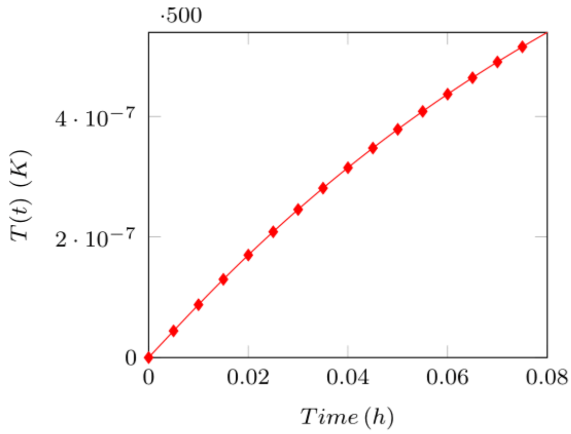
(c) Time versus $C_{Xe}(t)$



(d) Time versus $C_{Pm}(t)$



(e) Time versus $C_{Sm}(t)$



(f) Time versus $T(t)$

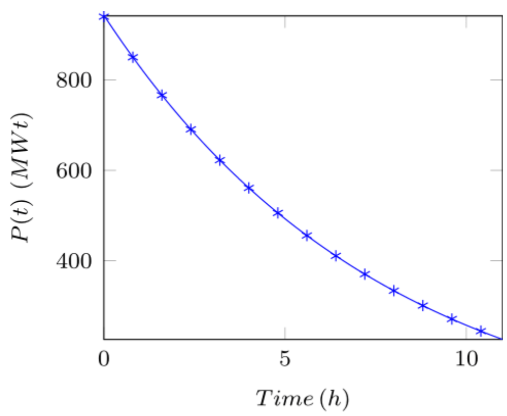
In the third interval, according to the reference of Silva [25], the reactor reaches a thermal power of only $30MWt$ caused by poisoning. In Fig. 4 item (a), it can be seen that the simulation, in this case, reached a value close to that of the accident. It is observed from items (c) and (e) in Fig. 4, that Xenon-135 reaches a value close to $5 \times 10^7 cm^{-3}$ in its concentration and Samarium-149, a value of $2 \times 10^6 cm^{-3}$. It can be seen that these values in the concentrations of these two elements remained nearly in this band throughout the intervals.

The fourth interval of the accident is characterized by the removal of the control rods and bringing the power back up to $200MWt$ in order to begin the safety test. Due to the negative reactivity caused by the neutron absorbers poisons the operators breached numerous safety regulations and procedures to try to achieve a power level in the range of $700MWt$ to $1000MWt$ necessary to perform the test. However, the combination of these errors triggered a cascade of events necessary to make the reactor unstable and on its way to the power excursion. From this point on, the test starts even with the protection and safety functions disabled.

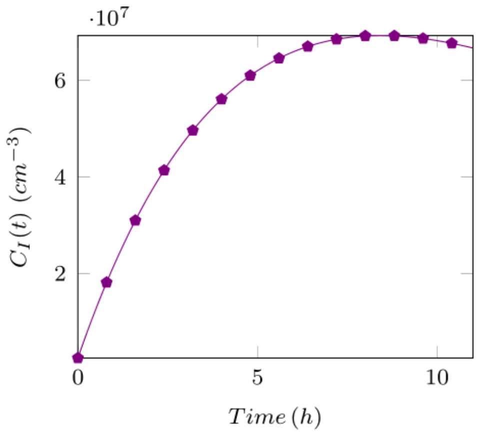
It can be seen from Fig. 5 item (a), that the power starts to increase, simulating the resumption of power due to the removal of the control rods for the start of the test. Due to the increase in neutron flux, it is observed that the concentration of Promethium-149, item (d), increases again. The same happens for element Iodine-135, as shown in item (b) of Fig. 5. The loss through radioactive decay of both Promethium-149 and Iodine-135 is not able to overcome its production through fission.

Similarly, the elements Xenon-135 and Samarium-149 maintain their growth, as observed in Fig. 5 and Fig. 6 items (c) and (e). The production rate of these elements becomes relatively higher than those of loss.

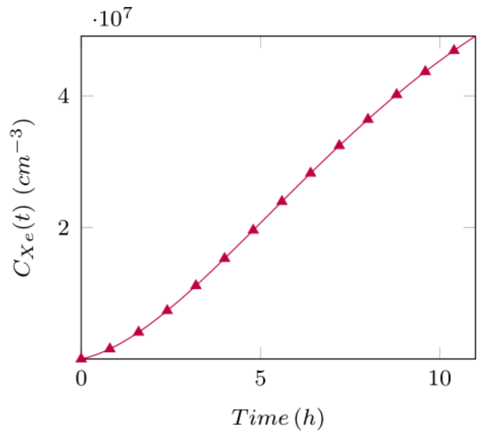
Figure 3: *Second interval of simulation (13:05 p.m. to 00:05 a.m.)*



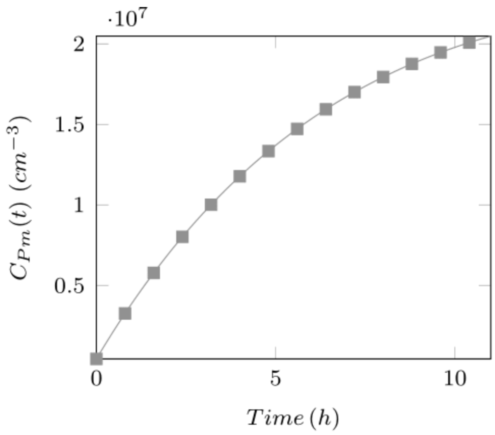
(a) *Time versus* $P(t)$



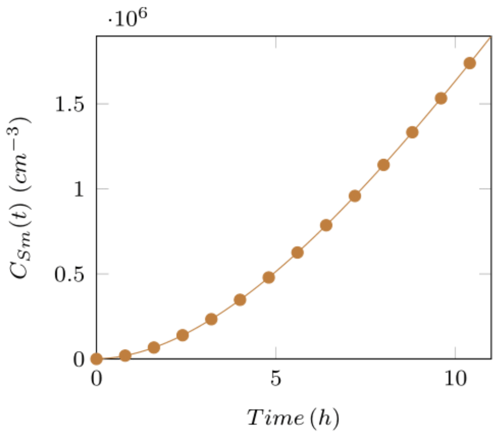
(b) Time versus $C_I(t)$



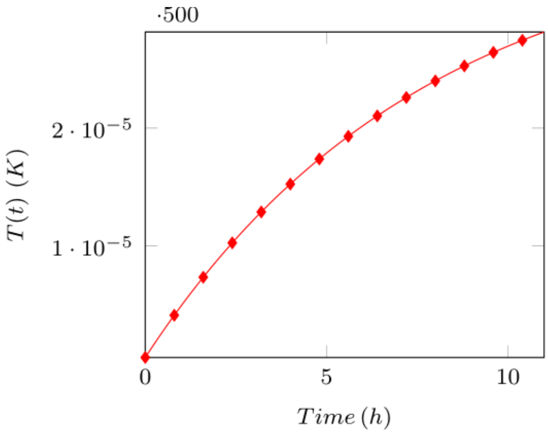
(c) Time versus $C_{Xe}(t)$



(d) Time versus $C_{Pm}(t)$

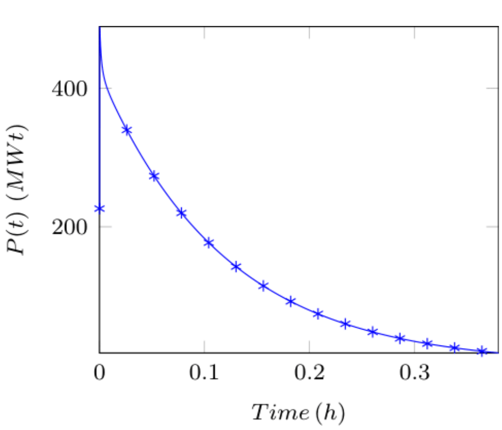


(e) Time versus $C_{Sm}(t)$

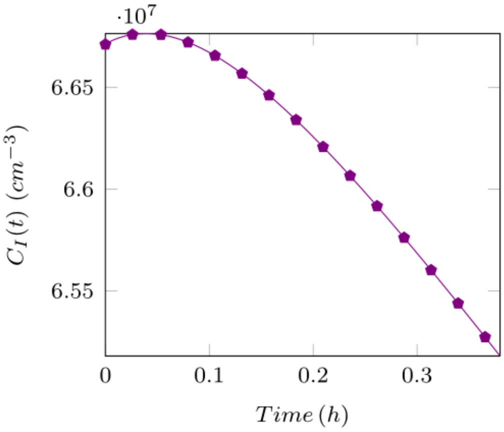


(f) Time versus $T(t)$

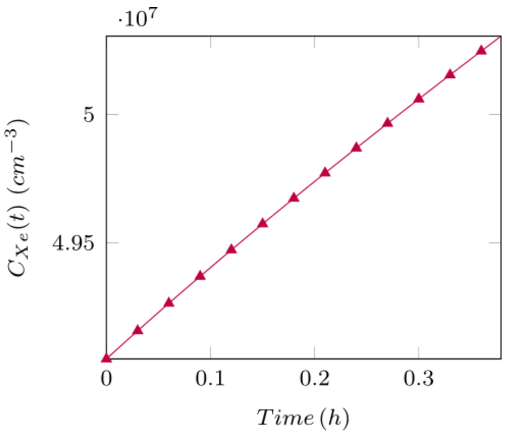
Figure 4: *Third interval of simulation (00:05 to 00:28 a.m.)*



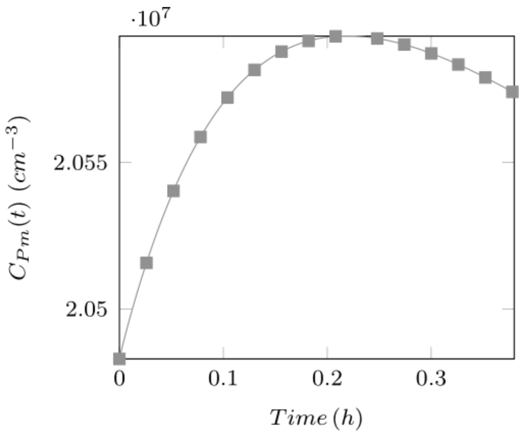
(a) *Time versus* $P(t)$



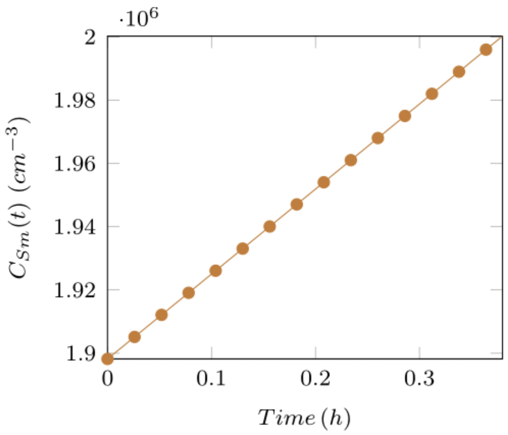
(b) Time versus $C_I(t)$



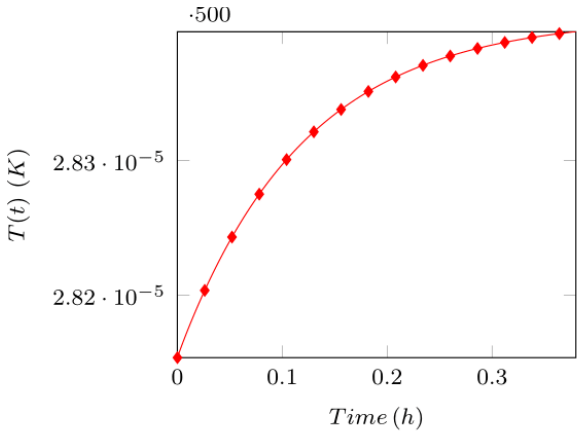
(c) Time versus $C_{Xe}(t)$



(d) Time versus $C_{Pm}(t)$

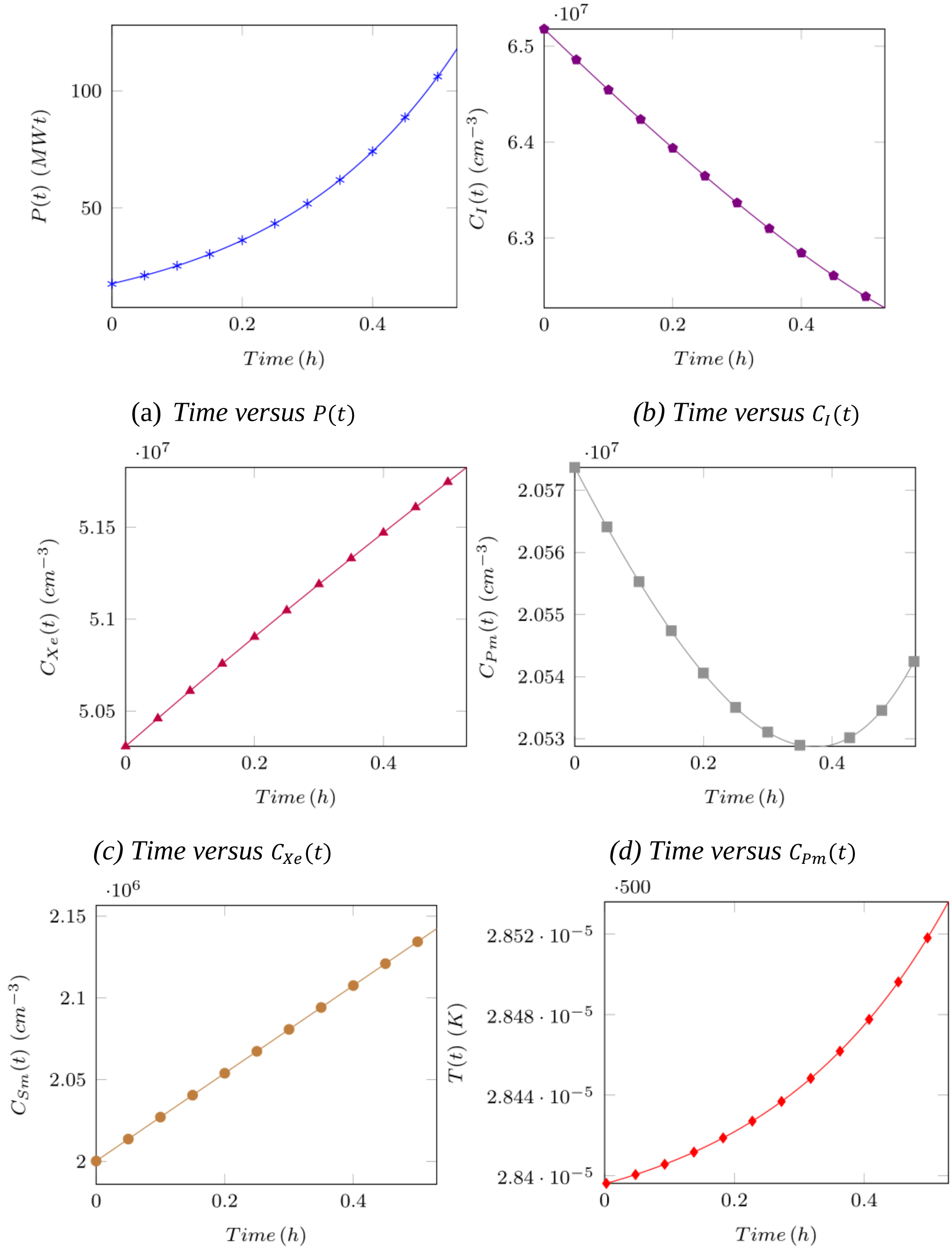


(e) Time versus $C_{Sm}(t)$



(f) Time versus $T(t)$

Figure 5: *Fourth interval of simulation (00:28 to 01:00 a.m.)*



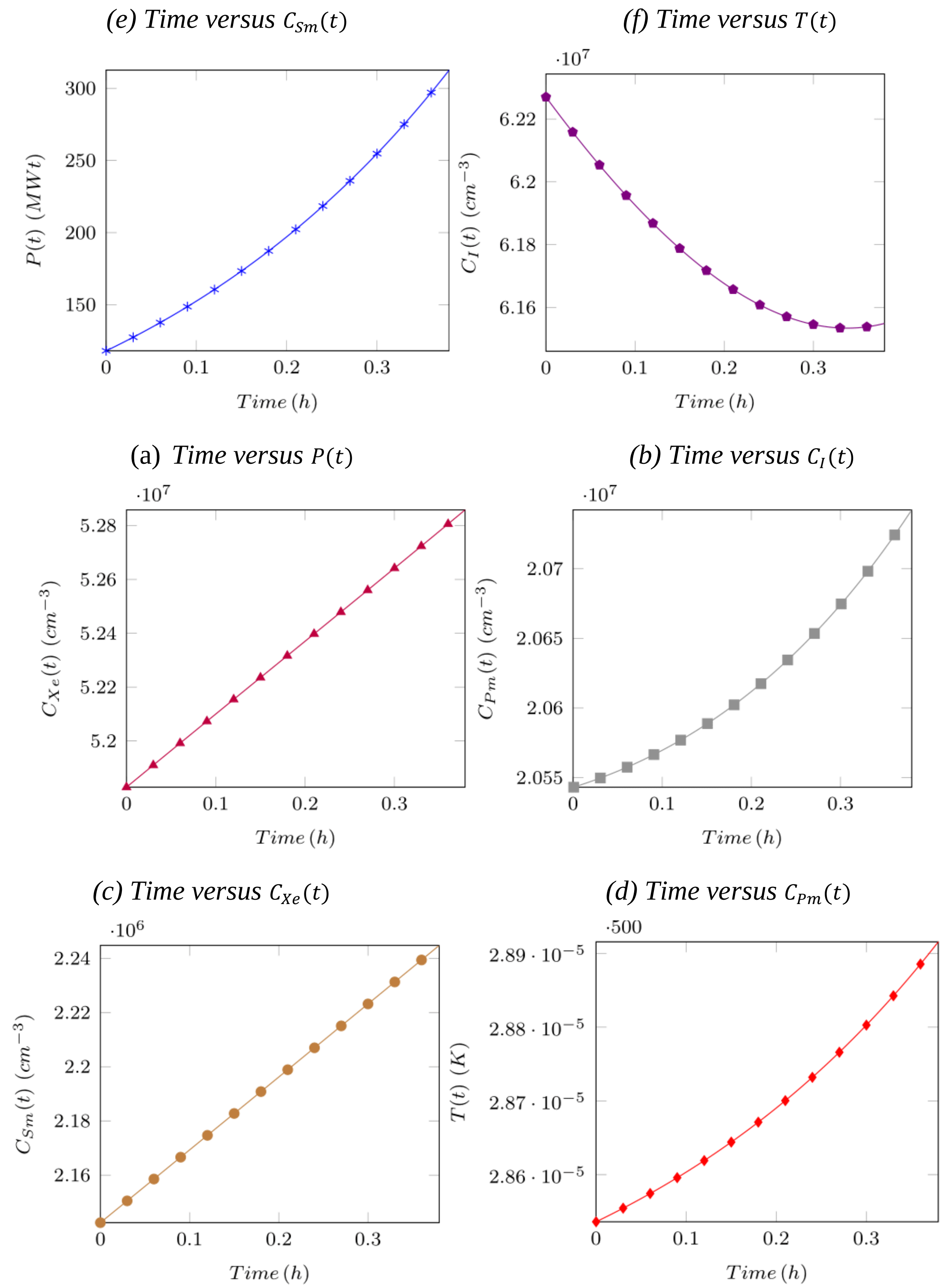
(a) *Time versus* $P(t)$

(b) Time versus $C_I(t)$

(c) Time versus $C_{Xe}(t)$

(d) Time versus $C_{Pm}(t)$

Figure 6: *Fifth interval of simulation (01:00 to 01:23:44 a.m.)*



(e) Time versus $C_{Sm}(t)$

(f) Time versus $T(t)$

(a) *Time versus* $P(t)$

(b) Time versus $C_I(t)$

(c) Time versus $C_{Xe}(t)$

(d) Time versus $C_{Pm}(t)$

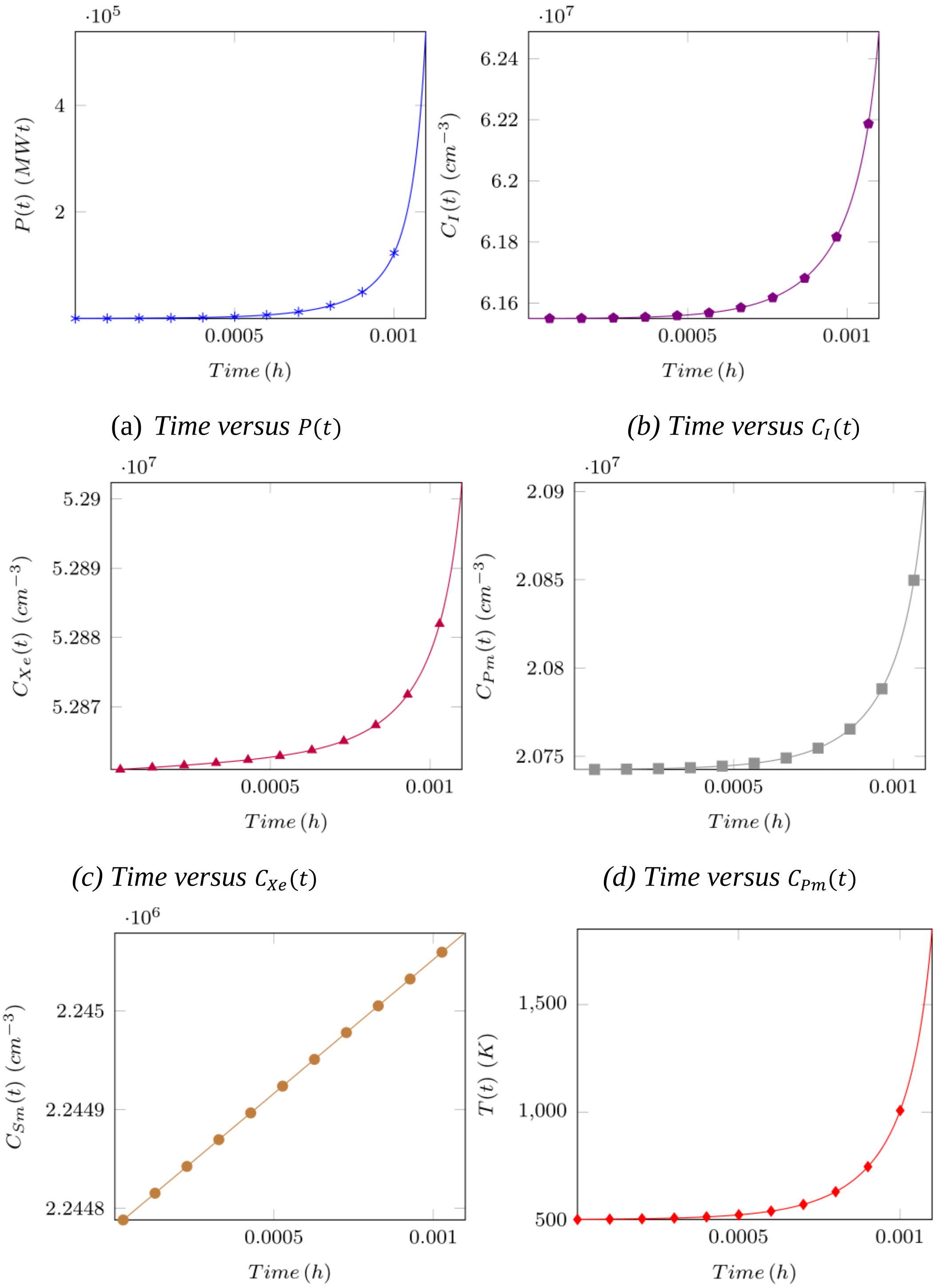
(e) Time versus $C_{Sm}(t)$ *(f) Time versus* $T(t)$

In the final seconds before the accident, described by interval VI, the reactor is practically out of control with power increasing rapidly. The emergency controls had been turned off and most of the control rods that serve to absorb neutrons and control nuclear fission had been removed. The closing of the main condenser valve due to the increase in water flow at the start of the test causes an increase in the positive reactivity coefficient. That is, the rise in temperature triggers an increase in steam, creating bubbles that prevent neutron absorption, further increasing power. As the neutron flux increases, the system itself automatically inserts the control rods, but it is not enough to compensate for the increase in reactivity caused by the increase in steam. Therefore, the operators press the AZ-5 button, which inserts all the control rods into the core, but due to the extreme heat, the fuel rods are deformed, preventing them from passing through. Then the power supply that drives the rods is cut off so that they could descend only driven by their own weight. However, at the tip of the control rods were graphite, which penetrated the unstable reactor core and ended up further increasing the power. From then on, it was only a matter of time before the core exploded. As the coolant flow was reduced, the heat and steam became uncontrollable, causing the pressure inside the core to exceed the level considered an accident level. Few seconds later the thermal explosion of the core occurred.

In Fletcher et al. [5], the average fuel temperature during the accident is presented, first held constant at $500K$, until the power excursion, exceeding $2000K$. In this last interval, since one does not specifically have the parameter H and due to the events that caused drastic changes in the reactor core, which may have altered the linearity relationship between temperature and density, the parameter was estimated from a more realistic approximation of the temperature in the power excursion, obtaining a value of $H = 33\,K/MWs$ and the temperature reaching about $2000K$ as shown in Fig. 7 item (f).

It is observed from item (a) of Fig. 7 that the power in the final interval, reached approximately $550000MWt$, coinciding with the value mentioned in Fletcher et al. [5]. Similarly, one also has an exponential growth in the concentrations of the neutron absorbers poisons in this last simulation interval.

Figure 7: *Sixth interval of simulation (01:23:44 to 01:23:48 a.m.)*



(a) *Time versus* $P(t)$

(b) Time versus $C_I(t)$

(c) Time versus $C_{Xe}(t)$

(d) Time versus $C_{Pm}(t)$

(e) Time versus $C_{Sm}(t)$

(f) Time versus $T(t)$

4. CONCLUSION

In this research, the results of the Neutron Point Kinetics model were presented by adding the effects of temperature and neutron absorbers poisons within a historical and technical context of the Chernobyl accident.

The simulation achieved the goal, showing the influence that absorbers poisons and temperature had on the power behavior in the Chernobyl accident. Thus, the importance of waiting the necessary time for the decay of fission product poisons due to the insertion effects of negative reactivity caused by poisoning was emphasized for the safe resumption of power. Furthermore, because the model does not include the thermal-hydraulic effects, the power values in some cases did not exactly match those found in the literature, but the results obtained are physically consistent with what was expected at each of the accident stages.

It is worth noting that the Point Kinetic model was able to extract coherent physical information to predict the reactor situation until the accident. It was also possible to verify that the Rosenbrock method produced coherent results, overcoming the degree of stiffness of the ODE system, besides solving a non-linear problem.

One of the contributing factors to the Chernobyl disaster was the effects of the absorbers poisons, in particular Xenon-135 because it has a high neutron absorption cross-section. The delayed response of reactor power to commands from the plant operators due to core poisoning led the operators to take the risky decision of removing practically all control rods from the system. Thus, numerous forbidden procedures were taken to bring the power back up to a certain value to perform a safety test. As a result, reactor instability was assured and the accident was only a matter of time.

Given the above, the motivation and highlight of this research was the analysis of the effects of temperature and neutron absorbers poisons on the final behavior of the power, bringing the accident at Chernobyl as the main focus. Even though the model considers only the variation of the amplitude of the neutron density with time, the goal was to obtain a physical idea, even if primary, of the reactor thermal power behavior, considering the effects of temperature and absorbers poisons at each stage of the accident. Understanding the effect of the negative reactivity caused by poisons and those of temperature on the final behavior of the power allows direct measures for the control and safe resumption of the power of a nuclear reactor.

It is worth pointing out that, to our knowledge, the approach taken to simulate the preliminary characteristics of the accident at Chernobyl is new to the literature, strongly emphasizing the importance of this study.

As prospects, it is intended to expand the idea proposed in this paper, using the model of Neutron Spatial Kinetics, which describes the temporal and spatial variation of the neutron population, by investigating other methods capable of solving this stiff and non-linear problem.

It is also intended, to simulate other hypothetical cases of the accident, such as a resumption of power after the decay time of the absorbers poisons.

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